New Frontiers of Attribute-Based Encryption via a General Paradigm and More

罗辑 (Ji Luo)

based on joint work with
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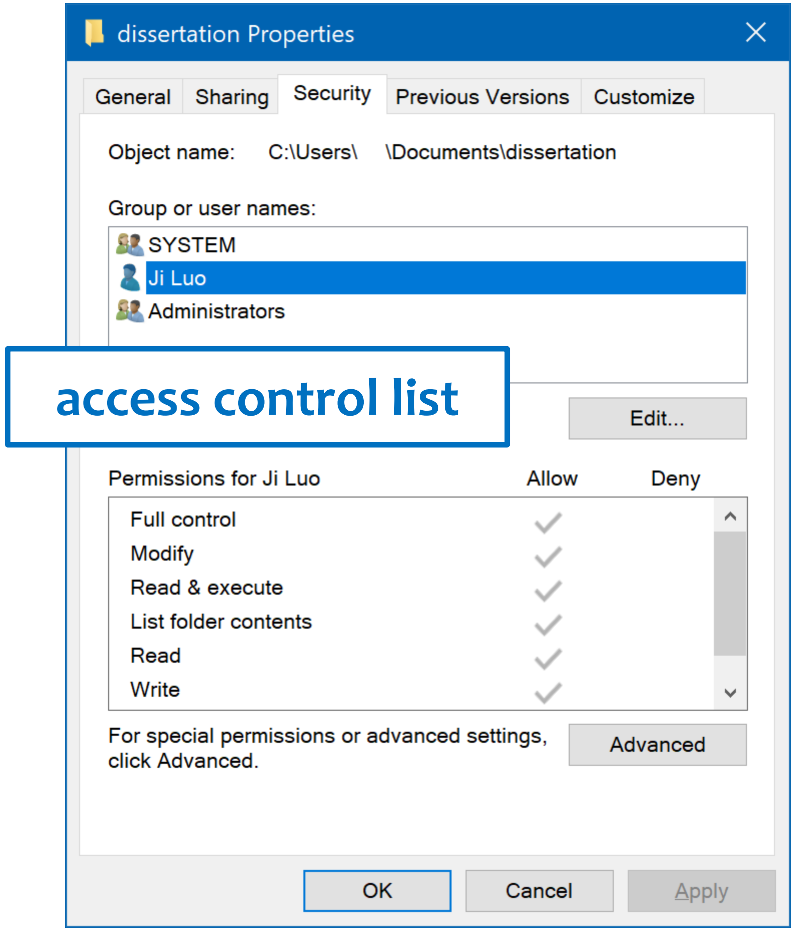
Attribute-Based Encryption [SW05,GPSW06]

= access control, enforced by cryptography



dissertation Properties

Object name: C:\Users\ \Documents\dissertation

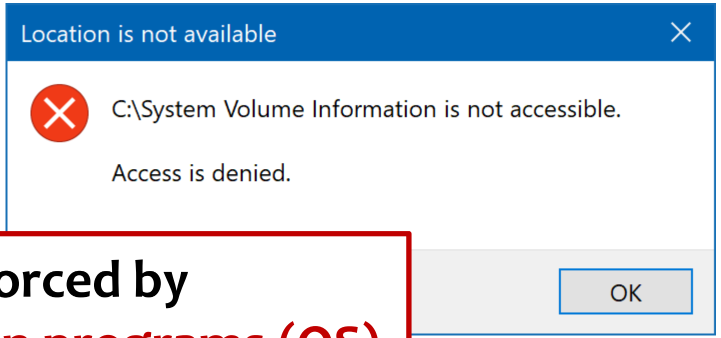
Group or user names:

- SYSTEM
- Ji Luo
- Administrators

Permissions for Ji Luo	Allow	Deny
Full control	✓	
Modify	✓	
Read & execute	✓	
List folder contents	✓	
Read	✓	
Write	✓	

For special permissions or advanced settings, click Advanced.

access control list



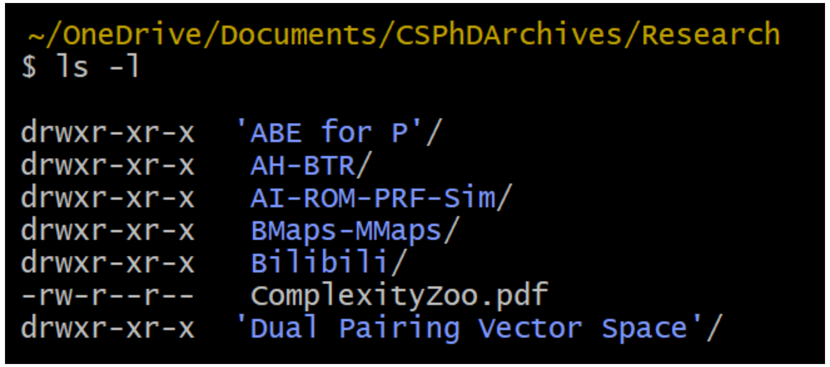
Location is not available

C:\System Volume Information is not accessible.

Access is denied.

only enforced by
checks in programs (OS)

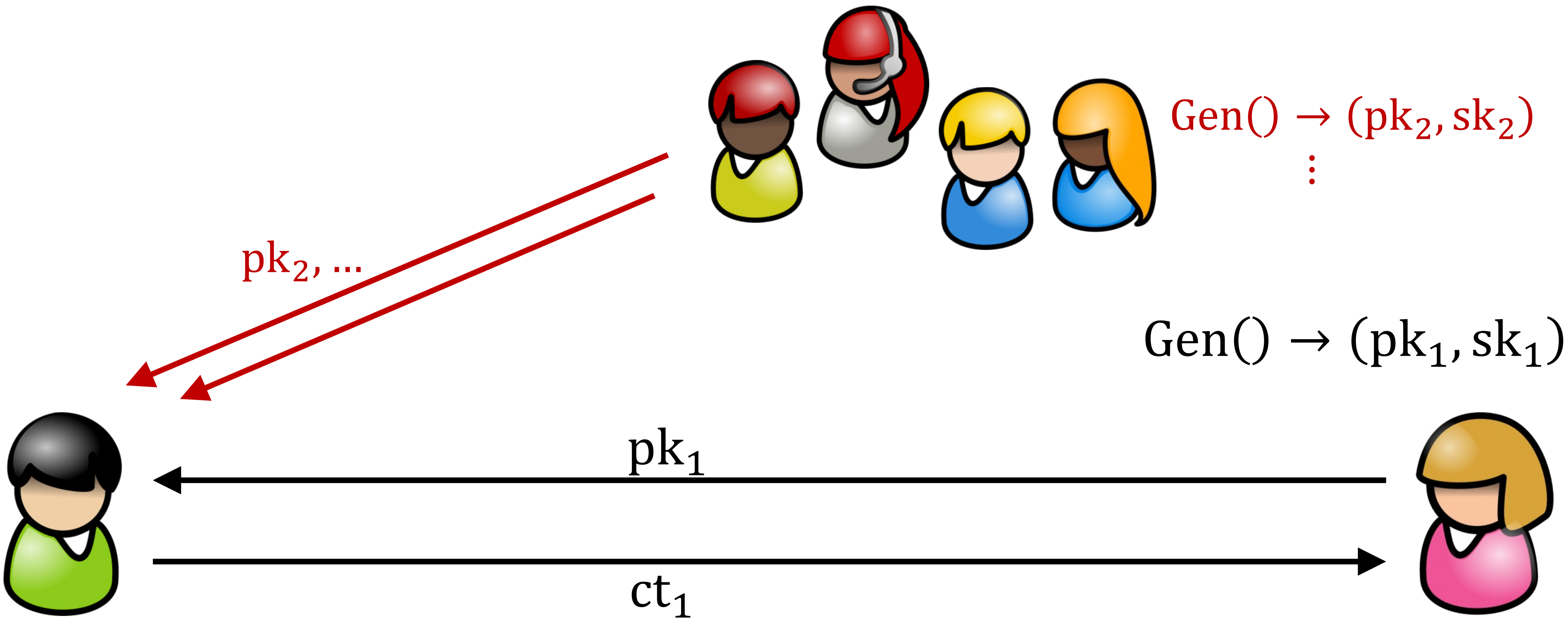
rwx rwx rwx
= permission bits



```
~/OneDrive/Documents/CSPhDArchives/Research
$ ls -l

drwxr-xr-x  'ABE for P'/
drwxr-xr-x   AH-BTR/
drwxr-xr-x   AI-ROM-PRF-Sim/
drwxr-xr-x   BMaps-MMaps/
drwxr-xr-x   Bilibili/
-rw-r--r--   ComplexityZoo.pdf
drwxr-xr-x  'Dual Pairing Vector Space'/
```

What about PKE for Access Control?



$\mathrm{Gen}() \rightarrow (\mathrm{pk}_2, \mathrm{sk}_2)$

$\vdots$

$\mathrm{pk}_2, \ldots$

$\mathrm{Gen}() \rightarrow (\mathrm{pk}_1, \mathrm{sk}_1)$

pk_1

ct_1

$\mathrm{Enc}(\mathrm{pk}_1, \mu) \rightarrow \mathrm{ct}_1$

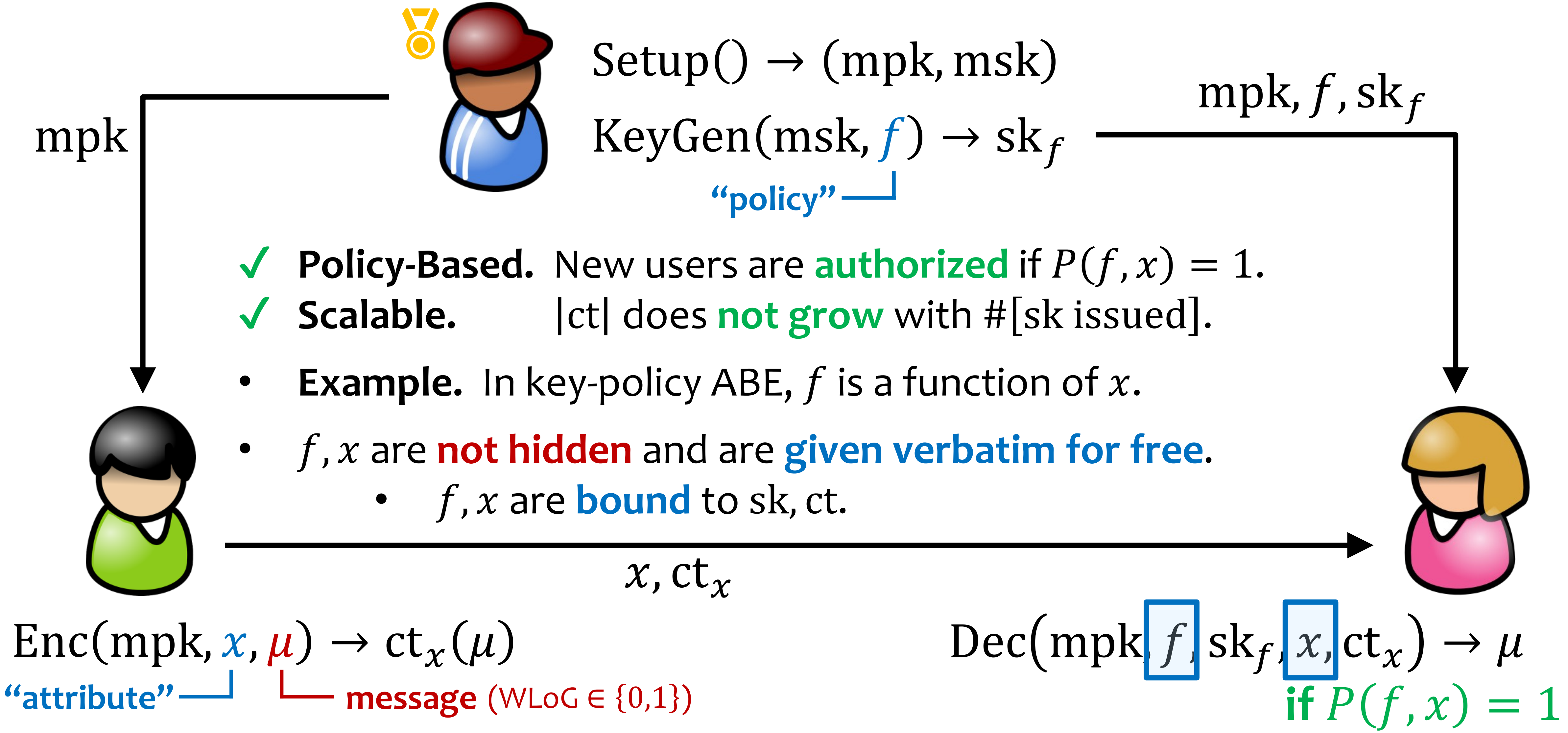
$\mathrm{Enc}(\mathrm{pk}_3, \mu) \rightarrow \mathrm{ct}_3$

$\vdots$

✗ new users always **un**authorized	✗ individual-based, not policy-based
✗ $\lvert\mathrm{ct}\rvert \propto \#[\mathrm{sk\ that\ can\ decrypt}]$	✗ not scalable

OS Analogy. New users (e.g., given suitable group membership) might be authorized.

Syntax and Correctness of ABE



$\mathsf{Setup}() \to (\mathsf{mpk}, \mathsf{msk})$

$\mathsf{KeyGen}(\mathsf{msk}, f) \to \mathsf{sk}_f$

"policy"

mpk

$\mathsf{mpk}, f, \mathsf{sk}_f$

✓ **Policy-Based.** New users are **authorized** if $P(f, x) = 1$.

✓ **Scalable.** $|\mathsf{ct}|$ does **not grow** with $\#[\mathsf{sk}\ \mathsf{issued}]$.

- **Example.** In key-policy ABE, f is a function of x.
- f, x are **not hidden** and are **given verbatim for free**.
  - f, x are **bound** to sk, ct.

x, ct_x

$\mathsf{Enc}(\mathsf{mpk}, x, \mu) \to \mathsf{ct}_x(\mu)$

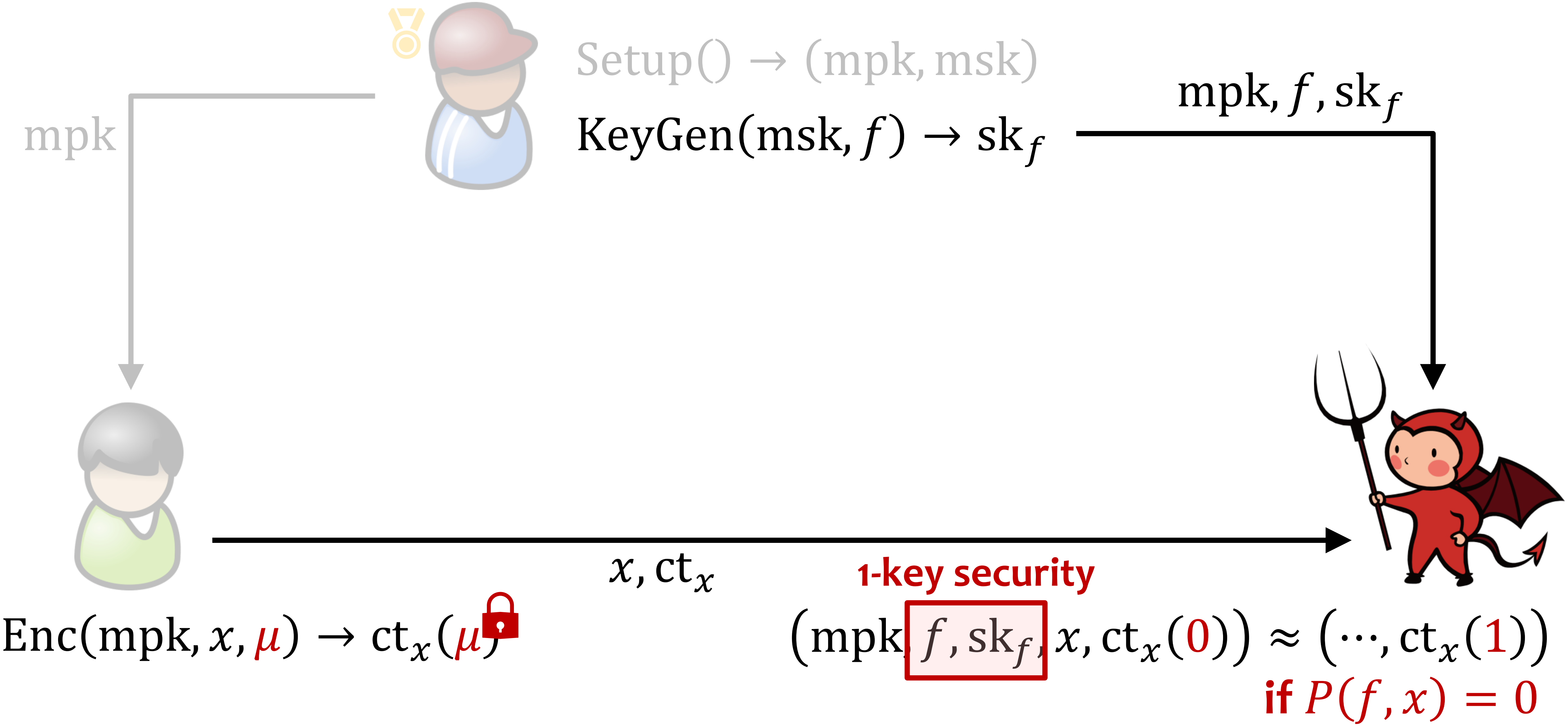
"attribute"

message (WLoG $\in \{0,1\}$)

$\mathsf{Dec}\big(\mathsf{mpk}, \boxed{f}, \mathsf{sk}_f, \boxed{x}, \mathsf{ct}_x\big) \to \mu$

if $P(f, x) = 1$

Security of ABE



$\text{Setup}() \to (\text{mpk}, \text{msk})$

$\text{KeyGen}(\text{msk}, f) \to \text{sk}_f$

mpk

$\text{mpk}, f, \text{sk}_f$

$\text{Enc}(\text{mpk}, x, \mu) \to \text{ct}_x(\mu)$ 🔒

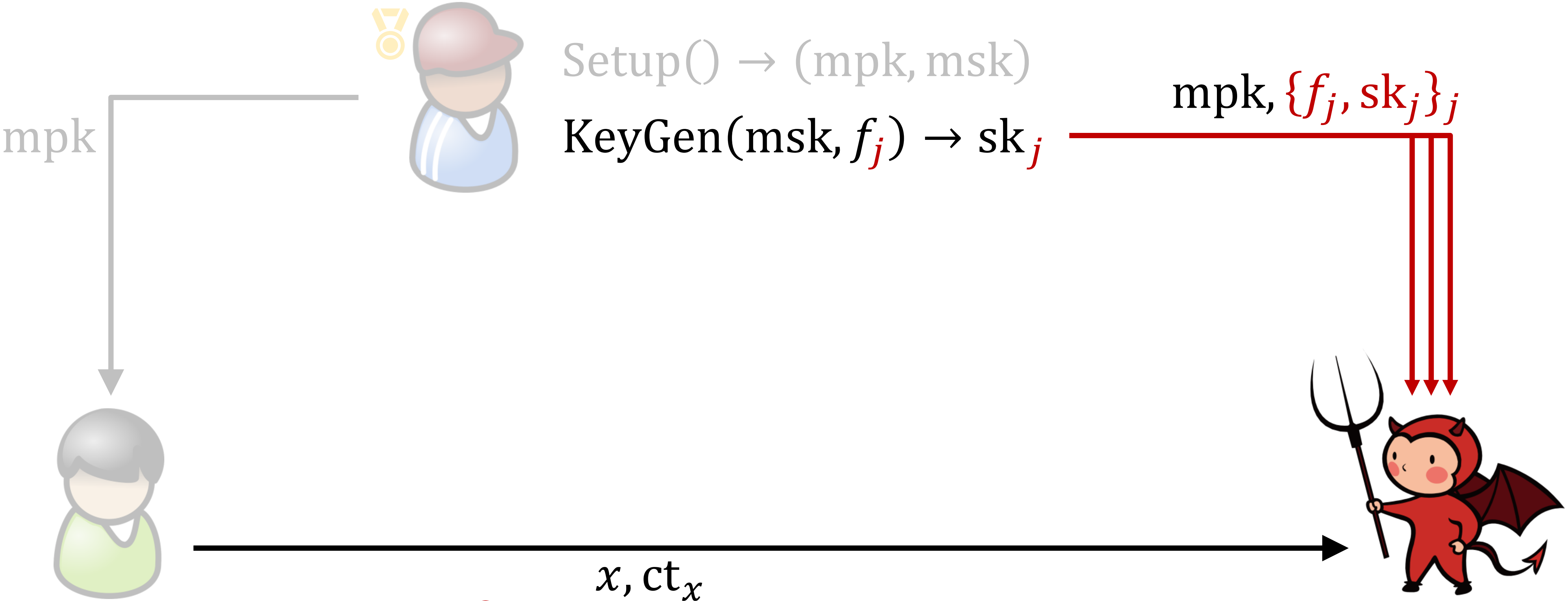
x, ct_x

1-key security

$$\left(\text{mpk}, \boxed{f, \text{sk}_f}, x, \text{ct}_x(0)\right) \approx \left(\cdots, \text{ct}_x(1)\right)$$

if $P(f, x) = 0$

Security of ABE – Collusion Resistance



$\mathsf{Setup}() \to (\mathsf{mpk}, \mathsf{msk})$

$\mathsf{KeyGen}(\mathsf{msk}, f_j) \to \mathsf{sk}_j$

$\mathsf{mpk}, \{f_j, \mathsf{sk}_j\}_j$

mpk

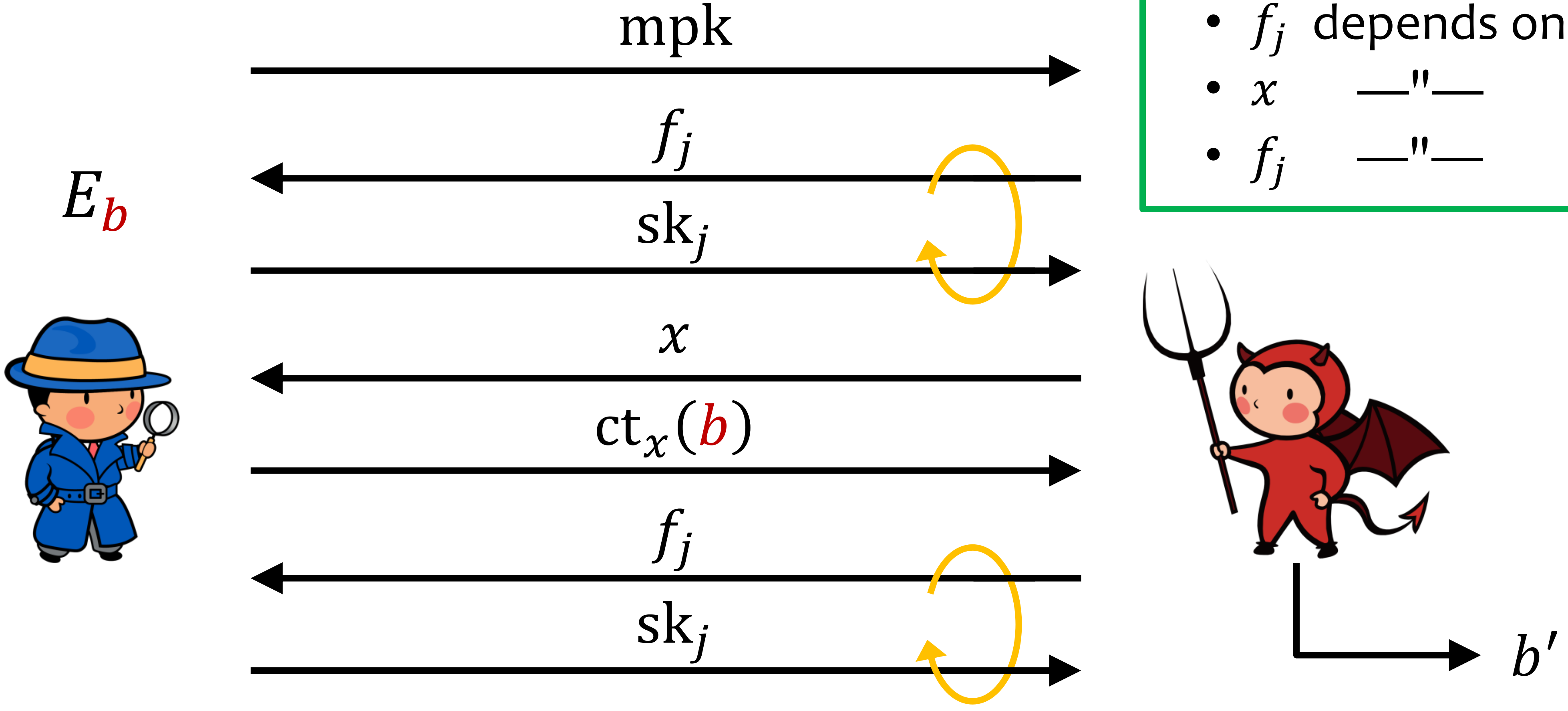
x, ct_x

$\mathsf{Enc}(\mathsf{mpk}, x, \mu) \to \mathsf{ct}_x(\mu)$ 🔒

$\left(\mathsf{mpk}, \{f_j, \mathsf{sk}_j\}_j, x, \mathsf{ct}_x(0)\right) \approx \left(\cdots, \mathsf{ct}_x(1)\right)$

if $P(f_j, x) = 0$ **for all** j

Security of ABE – Formal Definition

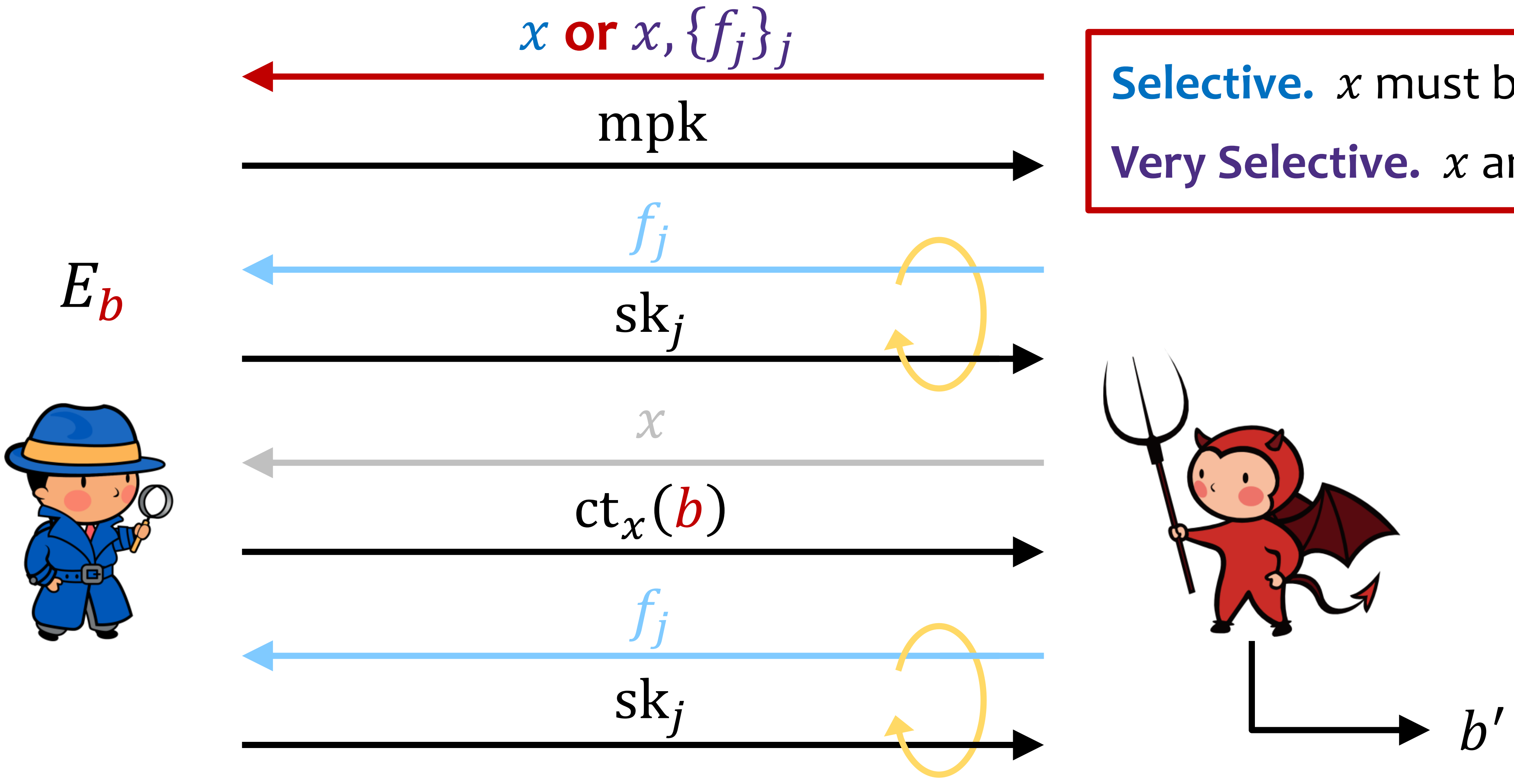


Adaptive Security.

- f_j depends on $\mathrm{mpk}, \mathrm{sk}_{<j}$
- x —"— $\mathrm{mpk}, \mathrm{sk}_{<J_1}$
- f_j —"— $\mathrm{mpk}, \mathrm{sk}_{<j}, \mathrm{ct}$

Security. $E_0 \approx E_1$ under the constraint of $P(f_j, x) = 0$ for all j.

Security of ABE – Weaker Notions

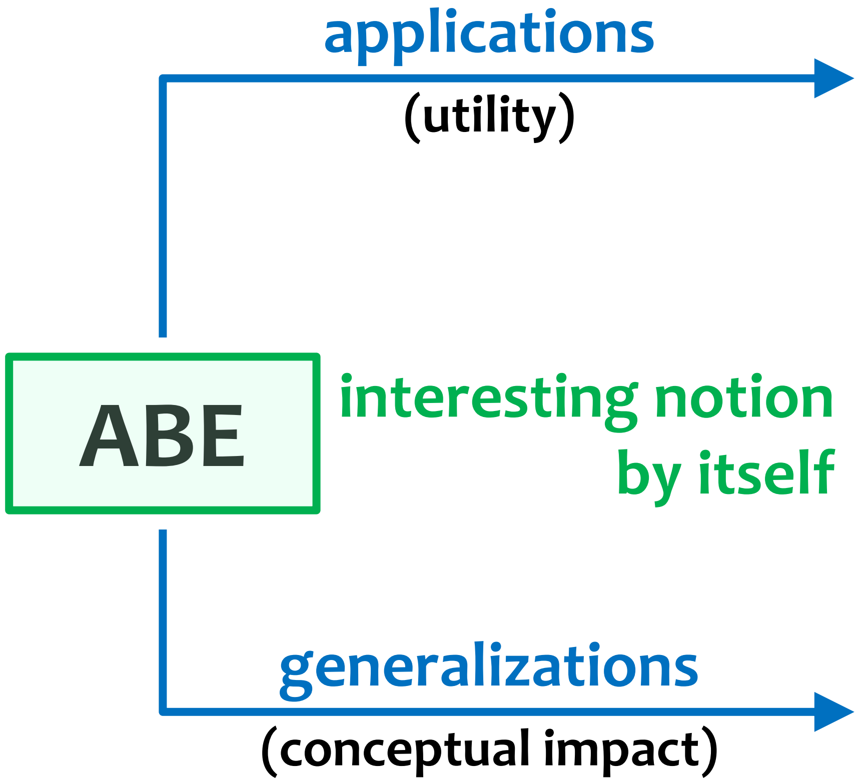


Selective. x must be chosen first.

Very Selective. x and all f_j's —"—.

Security. $E_0 \approx E_1$ under the constraint of $P(f_j, x) = 0$ for all j.

Why Study ABE?



applications

(utility)

ABE

interesting notion by itself

generalizations

(conceptual impact)

- access control

  audit logs [GPSW06] medical records [APGLPR11]

  private key distribution in cloud [Cloudflare17]

- verifiable delegated computation [PRV11]
- non-trivial witness encryption [BJKPW17]

- decentralization [C07,AYY22,HLWW22]

  multi-authority/input or registered

- stronger functionality [SBCSP07,BW07,BSW11]

  predicate / functional encryption

  ↑ connection to obfuscation [GGHRSW13,BV15,AJ15]

Pursuit of **Ends** – Desirata of ABE

Expressive. Supports rich class of policies.

circuits > formulae

RAM > TM > DFA

Succinct. Short mpk, sk, ct.

Recall $\mathrm{Dec}(\mathrm{mpk}, f, \boxed{\mathrm{sk}_f}, x, \boxed{\mathrm{ct}_x})$.

does not have to fully encode f, x

succinct

sk, ct **bound** to f, x (**not hiding**)

- think **hash / signature**
- possible that $|\mathrm{sk}| < |f|$, $|\mathrm{ct}| < |x|$

affects baseline

Efficient. Fast Dec (and Setup, KeyGen, Enc).

T_P = baseline for T_{Dec}

These objectives are intertwined 🪢 !

Strong Security. Adaptive > selective > very selective.

Weak Assumptions. Falsifiable > non-falsifiable.

Static > adversary-dependent (q-type).

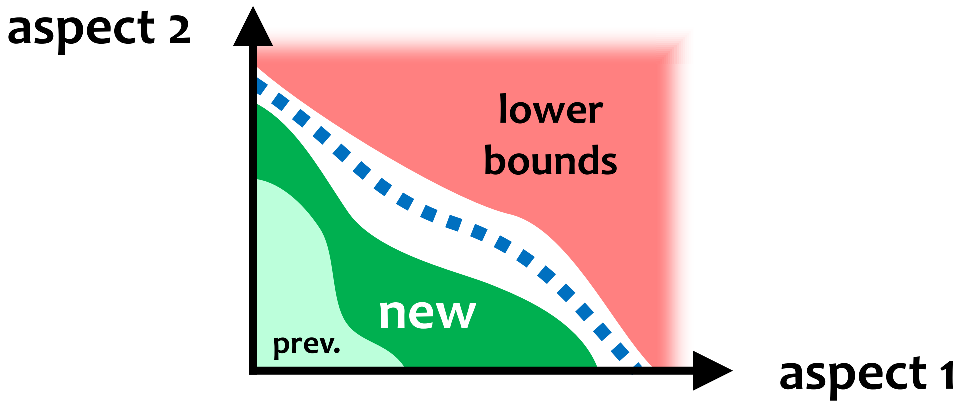
conceivable trade-off

- **same** construction
- proofs of **different** assumption ⇒ security

Pursuit of **Ends** – Multi-Objective Optimization

Expressive

Succinct



Goal. Characterize **curve of Pareto optimality**.

Efficient

Push the Frontier. Construct new schemes.

- **better** than previous in **at least one** aspect

  (wishful) better in **many** aspects

- some aspects are more prioritized

  (expressive, succinct)

Strong Security

Weak Assumptions

Encircle the Boundary. Prove trade-off lower bounds.

Pursuit of **Means**

Designing ABE schemes is… **not easy**!

general paradigm / framework ?

WANTED

- **modular** – redistribute complexities
- **powerful** – new results
- **versatile** – flexible assumptions

Previously…

dual system encryption [W09] + refinements

- pair encoding [A14]
- predicate encoding [W14]

❏ **born for adaptive security**
⚠ **only instantiated with pairing**
⚠ **heavy in algebra details**

two-to-one recoding [GVW13]
key-homomorphic encryption [BGGHNSVV14]

⚠ **new results only from lattices**
⚠ **too few instantiations**

Organization

Part I. General Paradigm (ABE $\Leftarrow$ IPFE $\circ$ Garbling)

- 4 instantiations
  [LL20a,LL20b,LLL22,HLL24]

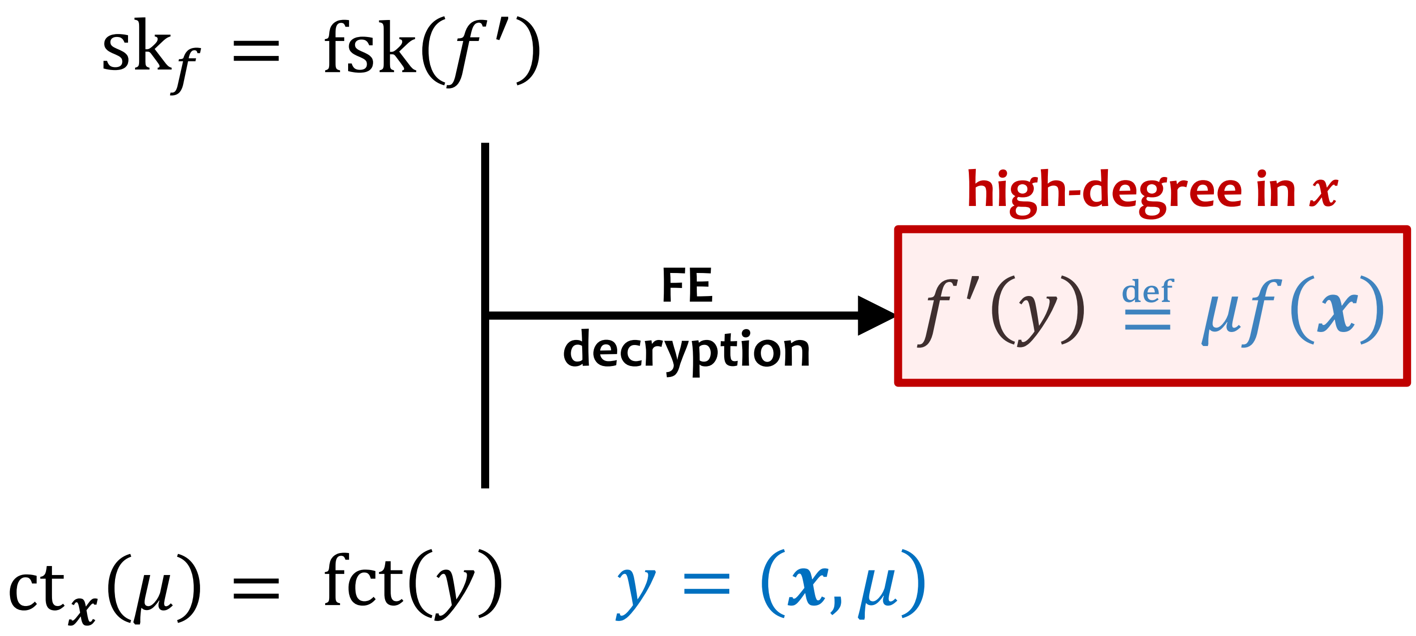
Part II. More

- ABE for circuits of unbounded depth from lattices
  [HLL23]
- first systematic study of
  optimal succinctness and efficiency for ABE
  [JLL23,L24]

Part I. **General Paradigm**

Somewhat technical, but less so than
the sum of all those separate talks.

ABE $\Leftarrow$ Functional Encryption



$\mathrm{sk}_f = \mathrm{fsk}(f')$

FE decryption

high-degree in $\boldsymbol{x}$

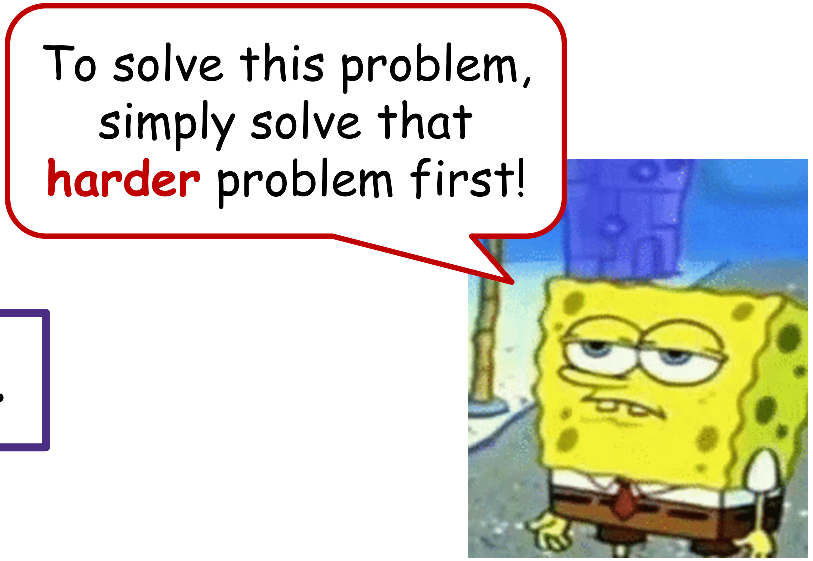
$f'(y) \overset{\text{def}}{=} \mu f(\boldsymbol{x})$

$\mathrm{ct}_{\boldsymbol{x}}(\mu) = \mathrm{fct}(y) \qquad y = (\boldsymbol{x}, \mu)$

Idea.

- Decompose into two phases (low-degree + high-degree).
- Use **FE** on **low-degree** only.

FE Security. Hides everything about y beyond $f'(y)$.


To solve this problem, simply solve that **harder** problem first!

Linear Garbling (Roughly) [Y82,Y86,AIK11,IW14]

1. $\mathrm{Garble}(f, \delta) \to (L_1, \dots, L_m)$
   - affine (low-degree) functions of x (**label functions**)
   - coefficients ($\boldsymbol{L}$'s) contain δ, randomness

Protect δ, randomness? Protect this process!

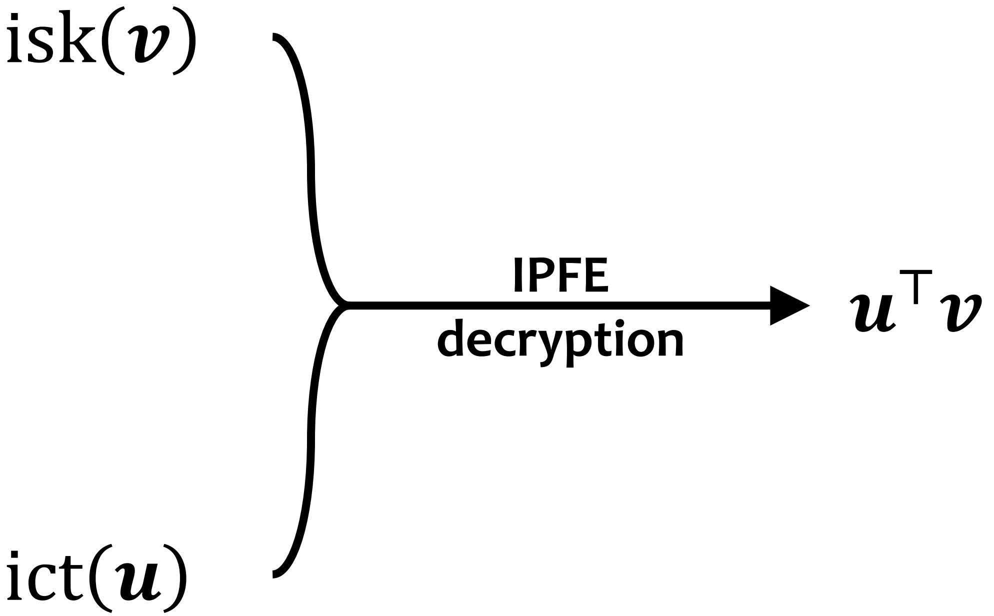
2. $\ell_1 = L_1(\boldsymbol{x}) = \boxed{\langle(1, \boldsymbol{x}), \boldsymbol{L}_1\rangle,}$ $\quad \dots, \quad$ $\ell_m = L_m(\boldsymbol{x}) = \boxed{\langle(1, \boldsymbol{x}), \boldsymbol{L}_m\rangle}$
   - **labels**

not hidden

3. $\mathrm{Eval}(f, \boxed{\boldsymbol{x},} \ell_1, \dots, \ell_m) \to \delta f(\boldsymbol{x})$
   - high-degree in $\boldsymbol{x}$

"ℓ's reveal nothing about δ beyond $\delta f(\boldsymbol{x})$"

Inner-Product FE (Roughly) [ABDP15]



$\mathrm{isk}(\boldsymbol{v})$

IPFE

decryption

$\boldsymbol{u}^\top \boldsymbol{v}$

$\mathrm{ict}(\boldsymbol{u})$

“isk, ict’s hide everything beyond the inner products”

ABE $\Leftarrow$ IPFE ∘ Garbling

$$\mathrm{sk}_f \;=\; \mathrm{isk}(\boldsymbol{L}_1)\,, \ldots, \mathrm{isk}(\boldsymbol{L}_m) \quad \text{(for } \delta\text{)}$$

$$\xrightarrow[\text{decryption}]{\text{IPFE}} \widehat{\delta f(\boldsymbol{x})} = L_1(\boldsymbol{x}), \ldots, L_m(\boldsymbol{x}) \xrightarrow[\text{evaluation}]{\text{garbling}} \delta f(\boldsymbol{x})$$

$$\mathrm{ct}_{\boldsymbol{x}}(\mu) \;=\; \mathrm{ict}(1, \boldsymbol{x}), \boxed{\delta \oplus \mu}$$

remove OTP
when $f(\boldsymbol{x}) = 1$

Composition of Security. (wishful)

- IPFE – only labels revealed
- garbling – only $\delta f(\boldsymbol{x})$ revealed
- δ is OTP for μ when $f(\boldsymbol{x}) = 0$

formalize properties that compose well

Security composition is tricky and sensitive to formalism.

Pairing Groups

- G_1, G_2, G_T groups of order p (prime)

  $G_i = \langle g_i \rangle$, additive, $[\![a]\!]_i \overset{\text{def}}{=} a g_i$

- $e: G_1 \times G_2 \to G_\mathrm{T}$ non-degenerate bilinear map

  $e(ag_1, bg_2) = abg_\mathrm{T}$, $[\![a]\!]_1 [\![b]\!]_2 = [\![ab]\!]_\mathrm{T}$

What is it good for cryptography?

Pairing = one-time, controlled multiplication.

✓ **Easy** $([\![a]\!]_1, b) \mapsto [\![ab]\!]_1$ and $([\![a]\!]_1, [\![b]\!]_2) \mapsto [\![ab]\!]_\mathrm{T}$.

DDH. $[\![a, b, ab]\!]_1 \approx [\![a, b, c]\!]_1$ for $a, b, c \overset{\$}{\leftarrow} \mathbb{Z}_p$.

✗ **Hard** $([\![a]\!]_1, [\![b]\!]_1) \mapsto [\![ab]\!]_\mathrm{T}$.

- Provides ***some*** protection for x in $[\![x]\!]_i$.
- Builds **IPFE** (*full* protection).

IPFE in [LL20a]

Recall. Garbling Eval after IPFE Dec.

Pairing-Based.

only linear operations with $[\![\cdot]\!]_{\mathrm{T}}$?

$$\mathrm{Dec}\big(\mathrm{isk}([\![\boldsymbol{v}]\!]_2),\ \mathrm{ict}([\![\boldsymbol{u}]\!]_1)\big) = [\![\boldsymbol{u}^\top \boldsymbol{v}]\!]_{\mathrm{T}}$$

Fact. Such IPFE can be built from k-Lin (**standard,** static assumption). [ALS16,W17,LV16,L17]

Function-Hiding.* (hides $\boldsymbol{u}, \boldsymbol{v}$)

$$\big(\mathrm{impk}, \{\mathrm{isk}(\boldsymbol{v}_{j0})\}_j, \{\mathrm{ict}(\boldsymbol{u}_{i0})\}_i\big) \approx \big(\mathrm{impk}, \{\mathrm{isk}(\boldsymbol{v}_{j1})\}_j, \{\mathrm{ict}(\boldsymbol{u}_{i1})\}_i\big)$$

Can compute $I \times J$ inner products $\boldsymbol{u}_{i?}^\top \boldsymbol{v}_{j?}$.

if $\boldsymbol{u}_{i0}^\top \boldsymbol{v}_{j0} = \boldsymbol{u}_{i1}^\top \boldsymbol{v}_{j1}$ for all i, j.

* not the full story, but good enough for now

Garbling in [LL20a]

More Linear Properties.

1. $\mathrm{Garble}(f, \delta; \boldsymbol{r}) \to (\boldsymbol{L}_1, \dots, \boldsymbol{L}_m)$
   linear in $(\delta, \boldsymbol{r})$
2. $\ell_j = \langle (1, \boldsymbol{x}), \boldsymbol{L}_j \rangle$
3. $\mathrm{Eval}(f, \boldsymbol{x}, \ell_1, \dots, \ell_m)$
   linear in $(\ell_1, \dots, \ell_m)$

Fact. Such garbling for arithmetic branching programs (ABP) exists. [IK00,IK02,IW14]

ABP = determinant of certain matrices

Security.* (distribution of $\ell_1, \dots, \ell_m$)

Point. This leads to **localized** label simulation.

1. $\ell_2, \dots, \ell_m$ are jointly random.
2. ℓ_1 is uniquely determined by $f, x, \delta f(\boldsymbol{x}), \ell_2, \dots, \ell_m$ due to **evaluation correctness**, i.e.,
   $$\mathrm{Eval}(f, x, \ell_1, \ell_2, \dots, \ell_m) = \delta f(\boldsymbol{x}),$$
   a linear constraint on ℓ_1.

* not the full story, but good enough for now

Instantiating the Paradigm in [LL20a]

$$\mathrm{sk}_f = \mathrm{isk}(\llbracket \boldsymbol{L}_1 \rrbracket_2), \ldots, \mathrm{isk}(\llbracket \boldsymbol{L}_m \rrbracket_2)$$

$$\xrightarrow[\textbf{decryption}]{\textbf{IPFE}} \widehat{\delta f(\boldsymbol{x})} = \llbracket \ell_1 \rrbracket_{\mathrm{T}}, \ldots, \llbracket \ell_m \rrbracket_{\mathrm{T}} \xrightarrow[\textbf{linear in } \ell\textbf{'s}]{\text{Eval}} \llbracket \delta f(\boldsymbol{x}) \rrbracket_{\mathrm{T}}$$

$$\mathrm{ct}_{\boldsymbol{x}}(\mu) = \mathrm{ict}(\llbracket 1, \boldsymbol{x} \rrbracket_1)$$

Selective Security in [LL20a]

✓ independent of δ

ict ($1, \boldsymbol{x}$)	IPFE $\approx$	ict ($1, \boldsymbol{x}$)	garbling $\equiv$	ict ($1, \boldsymbol{x}$)
isk ($\boldsymbol{L}_1$)		isk ($\ell_1, \mathbf{0}$)		isk ($\ell_1, \mathbf{0}$)
isk ($\boldsymbol{L}_2$)		isk ($\ell_2, \mathbf{0}$)		isk ($\ell_2, \mathbf{0}$)
$\vdots$		$\vdots$		$\vdots$
isk ($\boldsymbol{L}_m$)		isk ($\ell_m, \mathbf{0}$)		isk ($\ell_m, \mathbf{0}$)
		$\ell_j = \langle (1, \boldsymbol{x}), \boldsymbol{L}_j \rangle$		$\ell_2, \ldots, \ell_m = \$$

solve

$$\mathrm{Eval}(f, \boldsymbol{x}, \ell_1, \ell_2, \ldots, \ell_m) = \delta f(\boldsymbol{x})$$

for ℓ_1 $= 0$ **(constraint)**

Problem with Adaptive Security in [LL20a]

x not known at this point ↓

isk ($\boldsymbol{L}_1$)	isk ($\ell_1, \mathbf{0}$)	isk ($\ell_1, \mathbf{0}$)
isk ($\boldsymbol{L}_2$)	isk ($\ell_2, \mathbf{0}$)	isk ($\ell_2, \mathbf{0}$)
$\vdots$	$\vdots$	$\vdots$
isk ($\boldsymbol{L}_m$)	isk ($\ell_m, \mathbf{0}$)	isk ($\ell_m, \mathbf{0}$)
$\xrightarrow{\boldsymbol{x}}$	$\ell_j = \langle (1, \boldsymbol{x}), \boldsymbol{L}_j \rangle$	$\ell_2, \dots, \ell_m = \$$
ict ($1, \boldsymbol{x}$)	ict ($1, \boldsymbol{x}$)	ict ($1, \boldsymbol{x}$)

cannot solve equation dependent on x →

solve

$$\boxed{\mathrm{Eval}(f, \boldsymbol{x}, \ell_1, \ell_2, \dots, \ell_m) = 0}$$

for ℓ_1

Fixing Adaptive Security in [LL20a]

$\mathrm{isk}\,(\ \boldsymbol{L}_1 \quad 0\ )$

$\mathrm{isk}\,(\ \boldsymbol{L}_2 \quad 0\ )$

$\vdots$

$\mathrm{isk}\,(\ \boldsymbol{L}_m \quad 0\ )$

$\xrightarrow{\boldsymbol{x}}$

$\mathrm{ict}\,(\ 1, \boldsymbol{x} \quad 0\ )$

many steps *

$\approx$

✓ can be generated
✓ independent of δ

$\mathrm{isk}\,(\ 0, \mathbf{0} \quad 1\ )$

$\mathrm{isk}\,(\ \ell_2, \mathbf{0} \quad 0\ )$

$\vdots$

$\mathrm{isk}\,(\ \ell_m, \mathbf{0} \quad 0\ )$

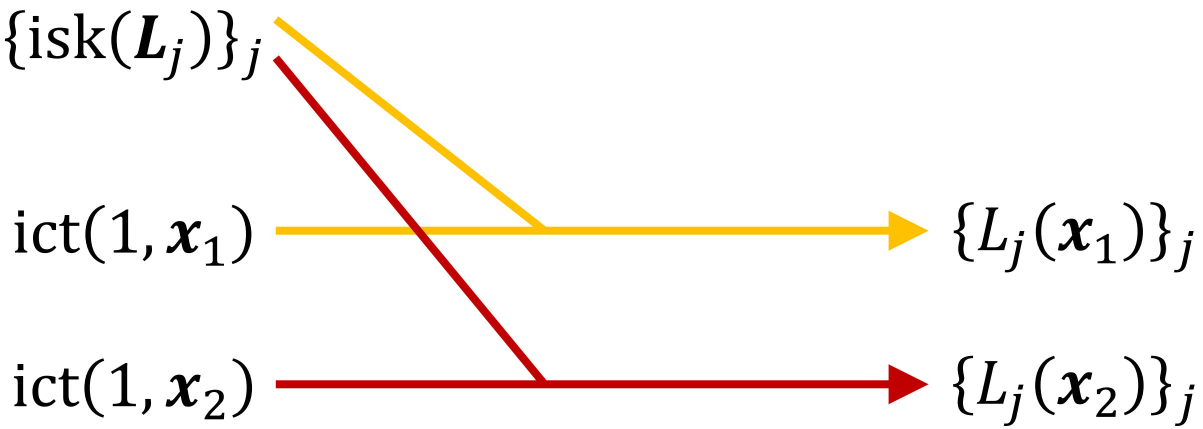
$\ell_2, \dots, \ell_m = \$$

$\mathrm{ict}\,(\ 1, \boldsymbol{x} \quad \ell_1\ )$

solve
$\mathrm{Eval}(f, \boldsymbol{x}, \ell_1, \ell_2, \dots, \ell_m) = 0$
for ℓ_1

* untold part of garbling security

Multi-Ciphertext Security in [LL20a]

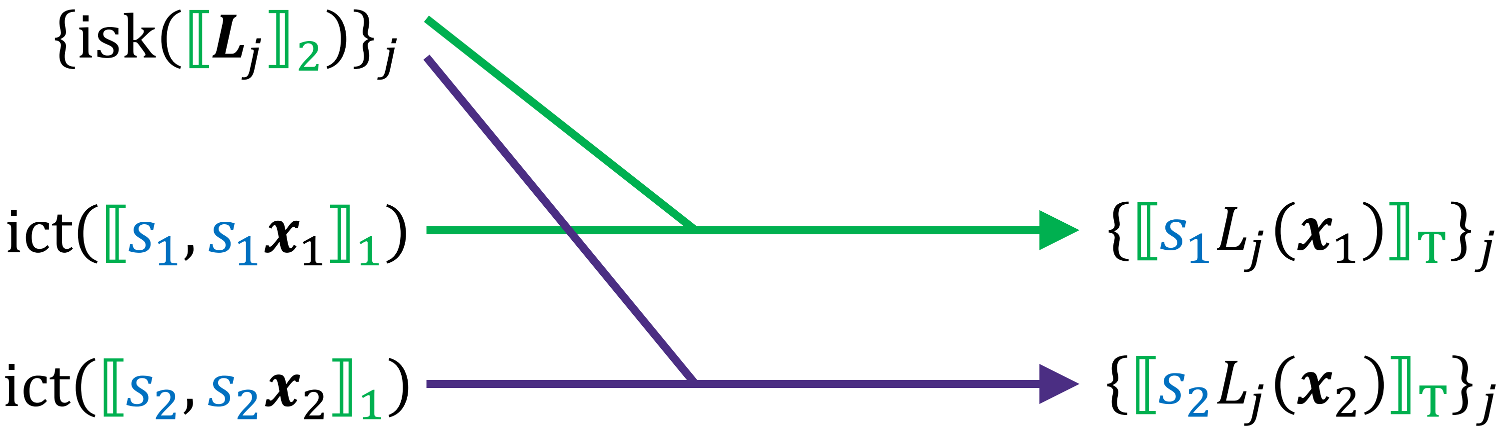


$\{\mathrm{isk}(\boldsymbol{L}_j)\}_j$

$\mathrm{ict}(1, \boldsymbol{x}_1)$ → $\{L_j(\boldsymbol{x}_1)\}_j$

$\mathrm{ict}(1, \boldsymbol{x}_2)$ → $\{L_j(\boldsymbol{x}_2)\}_j$

✗ Garbling security breaks if label functions are reused!



$\{\mathrm{isk}([\![\boldsymbol{L}_j]\!]_2)\}_j$

$\mathrm{ict}([\![s_1, s_1\boldsymbol{x}_1]\!]_1)$ → $\{[\![s_1 L_j(\boldsymbol{x}_1)]\!]_{\mathrm{T}}\}_j$

$\mathrm{ict}([\![s_2, s_2\boldsymbol{x}_2]\!]_1)$ → $\{[\![s_2 L_j(\boldsymbol{x}_2)]\!]_{\mathrm{T}}\}_j$

✓ DDH ensures $\{\boldsymbol{L}_j, \{s_i\boldsymbol{L}_j\}_i\}_j$ looks fresh in group.

ABE for Uniform in [LL20a]

Previous. input length of f is fixed (non-uniform model)

Now. more flexible (e.g., NFA)

- sk_Γ for regular expression Γ
  (works with ***all possible*** input length)
- $\mathrm{ct}_{\boldsymbol{x}}$ for input string $\boldsymbol{x}$
  (works with ***all possible*** reg.exp. size)

Same Paradigm.

- garbling for NFA, NL
- use IPFE to compute garbling
- proof guided by same idea

simple idea, complex execution,
IPFE helpful in managing proof

Tweaks. garbling size $\Theta(|\Gamma| \cdot |\boldsymbol{x}|)$

- sk = $\Theta(|\Gamma|)$ many isk's
- ct = $\Theta(|\boldsymbol{x}|)$ many ict's

make ***every pair*** of decryption useful!

Achievements of [LL20a]

ABE for Non-Uniform. ABP, adaptive, standard assumptions.

- **Previous.** puts bound on program size upon Setup [LOSTW10],
  or non-adaptive [GSPW06],
  or non-standard assumptions [LW12].
- **Previous, Concurrent.**
  for Boolean formula / branching programs [KW19,GW20].

ABE for Uniform. (N)L, (linear-size) N/DFA, adaptive, standard assumptions.

- **Previous.** for DFA,
  non-adaptive or large components or non-standard assumptions
  [W12,A14,AMY19,GWW19].
- **Concurrent.** [GW20]
  for DFA, same achievements;
  for NFA, non-adaptive.

* comparison only with pairing-based schemes

Power of Paradigm Exhibited by [LL20a]

one method solving **many** open problems (pairing-based)

- adaptive ABE for arithmetic computation / DFA
- ABE for NFA

almost the **end game** of adaptive standard ABE from pairing

- small remaining gap between selective/adaptive ABE
  (arithmetic **span** program vs A**B**P)
- **still the only** known adaptive ABE for NFA, L, NL
  (for ABP, improved in [LL20b])

✗ Size of garbling with our security notion is tightly related to ABP size. [Luo20汉]

reused in the **future**

- next-up in this talk
- same IPFE / garbling used for AB-FE for ABP, L [DP21,DPT22]

Remember Succinctness?

$|\mathrm{sk}_f| < |f|$?

$\mathrm{sk}_f = \mathrm{isk}(\boldsymbol{L}_1), \ldots, \mathrm{isk}(\boldsymbol{L}_m)$

IPFE

$\approx$

(x then f)

$\mathrm{isk}(\boldsymbol{\ell}_1, \mathbf{0}), \ldots, \mathrm{isk}(\boldsymbol{\ell}_m, \mathbf{0})$

- has $m = |f|$ objects (isk's)
- has $\geq m$ bits of (garbling) randomness

must hide garbling randomness

(non-hiding – more difficult for proof)

use non-hiding isk to *bind* to $\boldsymbol{x}$

$|\mathrm{ct}_x| < |\boldsymbol{x}|$?

$\mathrm{ct}_{\boldsymbol{x}}(\mu) = \mathrm{ict}(1, \boldsymbol{x})$

$\mathrm{ict}(1, \boldsymbol{x})$

- IPFE (hiding) security $\Longrightarrow$ $|\mathrm{ict}| \geq |\boldsymbol{x}|$

nothing to hide

no hiding required for "x then f" case

Using IPFE with Succinct Keys

$\mathrm{ct}_f(\mu) = \mathrm{ict}(\boldsymbol{L}_1), \ldots, \mathrm{ict}(\boldsymbol{L}_m)$

$\mathrm{ict}(0, \mathbf{0}, 1)$,
$\mathrm{ict}(\ell_2, \mathbf{0}), \ldots, \mathrm{ict}(\ell_m, \mathbf{0})$

many steps?

$\approx$

(f then x)

Two values hardwired during proof.

$\xrightarrow{\boldsymbol{x}}$

$\mathrm{sk}_{\boldsymbol{x}} = \mathrm{isk}(1, \boldsymbol{x})$

$\boxed{\mathrm{isk}(1, \boldsymbol{x}, \ell_1)}$

cannot hardwire ℓ_1
by changing vector

- ✓ $|\mathrm{isk}| = O(1)$
- ⚠ no hiding
- □ CP-1-ABE

Solution. IPFE with ***simulation security***.

(**stronger formulation compatible with proof**)

IPFE with Simulation Security

$$\begin{array}{ccc}
\mathrm{impk} & & \widetilde{\mathrm{impk}} \\[2em]
\{\, \mathrm{isk}(\ \boldsymbol{v}_j\ )\, \} & & \{\, \widetilde{\mathrm{isk}}\,(\overbrace{\ \boldsymbol{v}_j \ \big|\ \ \bot\ \ }^{\text{input to simulator}})\, \} \\[1em]
\mathrm{ict}(\ \boldsymbol{u}\ ) & \approx & \widetilde{\mathrm{ict}}\,(\ \bot\ \big|\ \{\boldsymbol{u}^\top \boldsymbol{v}_j\}_{j<J_1}\ ) \\[1em]
\{\, \mathrm{isk}(\ \boldsymbol{v}_j\ )\, \} & & \{\, \widetilde{\mathrm{isk}}\,(\ \boldsymbol{v}_j\ \big|\ \boldsymbol{u}^\top \boldsymbol{v}_j\ )\, \}
\end{array}$$

At every moment, (adaptive)
input to simulator is
whatever is **intended**
to be revealed.

Constructions. [LL20b]

- Generically from any selectively secure IPFE.
- Direct by modifying [ALS16] (better efficiency).

Stronger Formulation. [LL20b]

1. Can simulate up to T ciphertexts.
   (T tunable at Setup, affects component sizes)
2. Can **do/undo** simulation for any ict **in the presence of other** $\widetilde{\mathrm{ict}}$'s.

Using Simulation-Secure IPFE in [LL20b]

$$\mathrm{ct}_f(\mu) = \mathrm{ict}(\boldsymbol{L}_1), \ldots, \mathrm{ict}(\boldsymbol{L}_m)$$

$$\widetilde{\mathrm{ict}}(\bot|\bot),$$
$$\mathrm{ict}(\ell_2, \mathbf{0}), \ldots, \mathrm{ict}(\ell_m, \mathbf{0})$$

many steps

$\approx$

(f then x)

$\xrightarrow{\boldsymbol{x}}$

$$\mathrm{sk}_{\boldsymbol{x}} = \mathrm{isk}(1, \boldsymbol{x})$$

$$\widetilde{\mathrm{isk}}(1, \boldsymbol{x}|\ell_1)$$

✓ $|\mathrm{isk}| = O(T)$ with $T = 2$

❑ CP-1-ABE

Multi-key security? KP-ABE?

- CP-1-ABE + dual system [W09] ⟹ KP-ABE
- KP-ABE ⟹ KP-1-ABE (trivial)
- KP-1-ABE + dual system ⟹ CP-ABE*

* a factor of 2 shaved off in sizes compared to usual implementation of dual system, somehow…

Summary of [LL20b]

Achievements in Succinct ABE. ABP, adaptive, standard assumptions.

- **Part with x is Succinct.** ct_x in KP-ABE, sk_x in CP-ABE.
- **Previous.** only (natively) for Boolean computation,
  non-adaptive or non-standard assumptions [A16,ZGTCLQC16].
- **Concurrent.**
  for Boolean formulae [AT20]
  only 1 fewer group element in ct for KP.

What about the Paradigm? (not fully within paradigm)

- **"Ablation Study" of Roles of IPFE.** By comparing [LL20a] with literature…
  computing garbling (new IPFE);
  rerandomizing garbling (dual system).
- **Learn in Abstraction, Improve by Breaking It.**
  paradigm = bridge to reach the goal?

Moving Beyond Noiseless Garbling

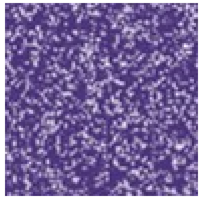
Part with x is Succinct. ct_x in KP-ABE, sk_x in CP-ABE.

Part with f?

Fact. **Size of noiseless linear garbling** tightly related to **span program size** [B84,M87,BDHM92,KW93] (linear algebraic computation, **low-depth**).

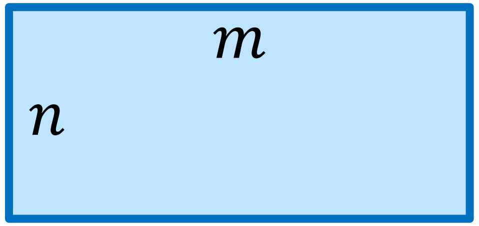
Noiseless

- cannot make f-part **succinct**
- does not handle **high** depth

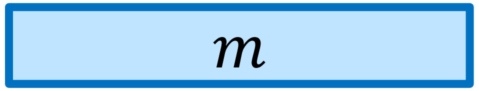


Let's try allowing noises!

Attribute Encoding from Lattices [BGGHNSVV14,GV15]



$$\boldsymbol{A} = (\boldsymbol{A}_1, \dots, \boldsymbol{A}_{|\boldsymbol{x}|}) \in \mathbb{Z}_q^{n\times|\boldsymbol{x}|m} \xrightarrow[\text{for circuit } C]{\text{EvalC}} \boldsymbol{A}_C \in \mathbb{Z}_q^{n\times m}$$



$$\boldsymbol{s}^\top(\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G}) + \boldsymbol{e}^\top \xrightarrow[\text{for } C \text{ and } \boldsymbol{x}]{\text{EvalCX}} \boldsymbol{s}^\top(\boldsymbol{A}_C - C(\boldsymbol{x}) \cdot \boldsymbol{G}) + \boldsymbol{e}_C^\top$$

$$\uparrow$$

$$= \left(\boldsymbol{s}^\top(\boldsymbol{A}_1 - \boldsymbol{x}[1] \cdot \boldsymbol{G}) + \boldsymbol{e}_1^\top, \dots\right)$$

- **homomorphic** encoding
- sizes depend on depth d of C, **not size**
- noise growth is **exponential** in d

*** What is $\boldsymbol{G}$?**
Some fixed, publicly known matrix – details not needed for now.

Noisy Linear Garbling from Attribute Encoding

Public Parameters. $\boldsymbol{A}$, short $\boldsymbol{z}$

Think binary x.

Labels. $\boldsymbol{s}^\top(\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G}) + \boldsymbol{e}^\top = \utilde{\boldsymbol{s}^\top(\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G})} = \boldsymbol{c}^\top$ (wavy = noises)

Evaluation.

1. $\boldsymbol{c}^\top \xrightarrow{\text{EvalCX}} \utilde{\boldsymbol{s}^\top(\boldsymbol{A}_C - C(\boldsymbol{x}) \cdot \boldsymbol{G})} = \boldsymbol{c}_C^\top$
2. output $\boldsymbol{c}_C^\top \boldsymbol{z}$

$$= \utilde{\boldsymbol{s}^\top \boldsymbol{A}_C \boldsymbol{z}} - C(\boldsymbol{x}) \cdot \utilde{\boldsymbol{s}^\top \boldsymbol{G} \boldsymbol{z}}$$

- $C(\boldsymbol{x}) = 0$, then just the secret
- $C(\boldsymbol{x}) = 1$, then $\utilde{\boldsymbol{s}^\top \boldsymbol{G} \boldsymbol{z}}$ is OTP to hide secret *

Changes.

- "$P(C, x) = \neg C(\boldsymbol{x})$" – recover secret when $C(\boldsymbol{x}) = 0$
- secret is $\utilde{\boldsymbol{s}^\top \boldsymbol{A}_C \boldsymbol{z}}$

* not the full story, but good enough for now

Using Noisy Linear Garbling in [LLL22]

$|\mathsf{ct}_f| = O(|\boldsymbol{x}|^2) < |f|$

$$\mathsf{ct}_f(\mu) = \mathsf{ict}([\![\boldsymbol{s}^\top \boldsymbol{A}_1 + \boldsymbol{e}_1^\top,\ \boldsymbol{s}^\top(\boldsymbol{A}_1 - \boldsymbol{G}) + \boldsymbol{e}_1^\top]\!]_1), \ldots$$

✓ polynomial noise for log-depth circuits (formulae)

$$\xrightarrow[\text{decryption}]{\text{IPFE}} \boxed{[\![r(}\boldsymbol{s}^\top(\boldsymbol{A}_1 - \boldsymbol{x}[1]\cdot\boldsymbol{G}) + \boxed{\boldsymbol{e}_1^\top)}]\!]_{\mathrm{T}}, \ldots \xrightarrow{\text{Eval}} [\![\mu]\!]_{\mathrm{T}}$$

DDH cannot rerandomize *small noises*

generic group model + stronger IPFE security

$|\mathsf{sk}_x| = O(1)$

$$\mathsf{sk}_{\boldsymbol{x}} = \mathsf{isk}(r[\![\neg\boldsymbol{x}, \boldsymbol{x}]\!]_2)$$

- selects $\boldsymbol{s}^\top\boldsymbol{A}_1$ or $\boldsymbol{s}^\top(\boldsymbol{A}_1 - \boldsymbol{G})$ etc.
- DDH-style rerandomization with r

Generic Group Model [S97,M05]

Standard Model.

- arbitrary computation on group element represented in bits
- certain computational problem is hard

Generic Group Model. intuitive although strong

- only operations via group-theoretic interfaces
  - addition, negation, zero-testing
  - pairing
- (equivalently) adversary capability

  = zero-test any **linear** function of $(\llbracket 1, \boldsymbol{w}_1 \rrbracket_1 \otimes \llbracket 1, \boldsymbol{w}_2 \rrbracket_2, \llbracket \boldsymbol{w}_{\mathrm{T}} \rrbracket_{\mathrm{T}})$

more control of adversarial behavior
$\Longrightarrow$ easier to write proofs

Saw $w_1 = \llbracket a \rrbracket_1, w_2 = \llbracket b \rrbracket_2, w_{\mathrm{T}} = \llbracket c \rrbracket_{\mathrm{T}}$.
Define $L(r, s) = r - s$. Ask $L(\boldsymbol{w}_1 \otimes \boldsymbol{w}_2, \boldsymbol{w}_{\mathrm{T}}) \overset{?}{=} 0$.

Yes/no. (Tests whether $ab = c$.)

Very Strong IPFE Simulation Security of [LLL22]

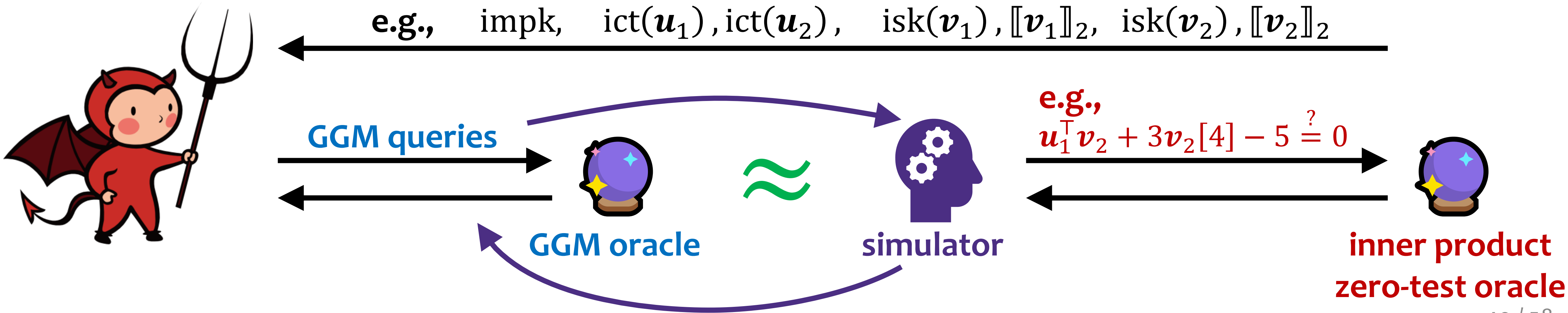
Generic Group Model.

> *"All you can do is to **zero-test** linear functions of **pairing results** (and target group elements)."*

IPFE Simulation in GGM.

> *"All you can do is to **zero-test** linear functions of **inner products** (and key vectors)."*

✓ proven for [ABDP15]
✓ rerandomization with r now **works**



e.g., $\mathrm{impk}, \quad \mathrm{ict}(\boldsymbol{u}_1), \mathrm{ict}(\boldsymbol{u}_2), \quad \mathrm{isk}(\boldsymbol{v}_1), [\![\boldsymbol{v}_1]\!]_2, \quad \mathrm{isk}(\boldsymbol{v}_2), [\![\boldsymbol{v}_2]\!]_2$

e.g., $\boldsymbol{u}_1^{\top}\boldsymbol{v}_2 + 3\boldsymbol{v}_2[4] - 5 \overset{?}{=} 0$

Summary of [LLL22]

Doubly Succinct CP-ABE. for Boolean **formulae** (log-depth circuits)

- $|\mathrm{sk}_{\boldsymbol{x}}| = O(1) < |\boldsymbol{x}|$ and $|\mathrm{ct}_f| = O(|\boldsymbol{x}|^2) < |f|$
- **first** ABE with non-trivial **double** succinctness

- **Previous.** [AY20] CP-ABE from pairing + lattices
  [AWY20] —"— with [LL20a] IPFE (not **doubly** succinct)

More Succinct KP-ABE. for Boolean **circuits** with $|\mathrm{sk}_C| = O(1)$

- **Previous.** [BGGHNSVV14] with $|\mathrm{sk}_C| = \mathrm{poly}(d)$
- **Later.** [CW23] from just LWE

Versality of Paradigm. Can **combine** pairing and lattices.

Lattices, not Pairing

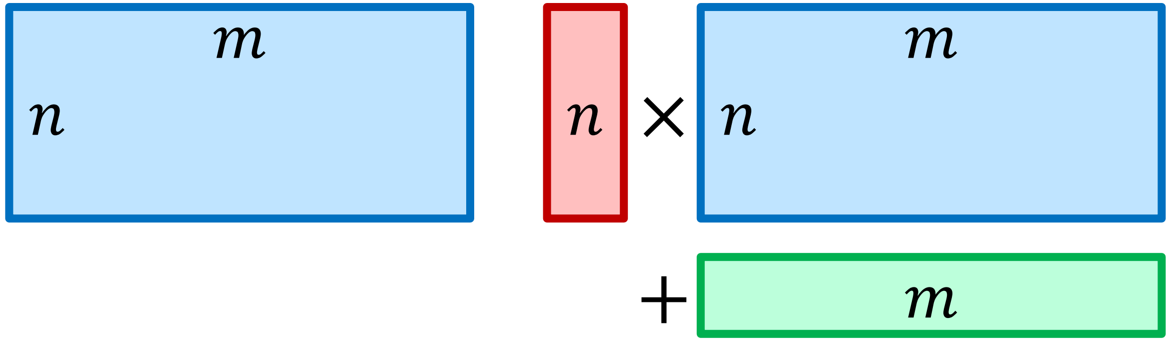
Why not pairing?

- not post-quantum secure
- noise must be polynomially bounded

Learning with Errors (LWE).

protection for $\boldsymbol{s}$

$$\boldsymbol{A}, \qquad \boldsymbol{s}^\top \boldsymbol{A} + \boldsymbol{e}^\top \qquad \approx \qquad \boldsymbol{A}, \ \$$$



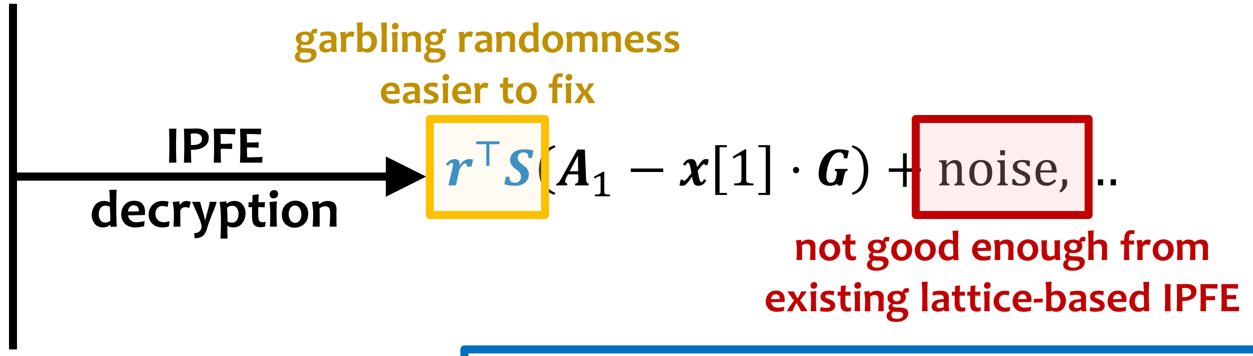
- presumably post-quantum
- OK with somewhat large noise

✓ builds some IPFE

✗ IPFE insufficient for ABE

Rerandomization with Lattice-Based IPFE

$$\mathrm{ct}_f(\mu) = \mathrm{ict}(\boldsymbol{S}\boldsymbol{A}_1,\ \boldsymbol{S}(\boldsymbol{A}_1 - \boldsymbol{G}))\,, \ldots$$



IPFE decryption

garbling randomness easier to fix

$$\boxed{\boldsymbol{r}^\top\boldsymbol{S}}(\boldsymbol{A}_1 - \boldsymbol{x}[1]\cdot\boldsymbol{G}) + \boxed{\text{noise},}\ \ldots$$

not good enough from existing lattice-based IPFE

$$\mathrm{sk}_{\boldsymbol{x}} = \mathrm{isk}(\boldsymbol{r}^\top(\neg\boldsymbol{x}, \boldsymbol{x}))$$

Goal. lattice-based IPFE giving good noises

Lattice Trapdoors [MP12] and Evasive LWE [W22,T22]

“$\boldsymbol{K} = \boldsymbol{B}^{-1}(\boldsymbol{P})$”

trapdoor of $\boldsymbol{B}$ = information about $\boldsymbol{B}$ for solving $\boldsymbol{B}\boldsymbol{K} = \boldsymbol{P}$ for **small** $\boldsymbol{K}$, given any $\boldsymbol{P}$

$$(\boldsymbol{s}^\top \boldsymbol{B} + \boldsymbol{e}^\top) \cdot \boldsymbol{B}^{-1}(\boldsymbol{P}) = \boldsymbol{s}^\top \boldsymbol{P} + \boldsymbol{e}^\top \boldsymbol{K}$$

$$\boxed{\boldsymbol{s}^\top \boldsymbol{B} \cdot \boldsymbol{B}^{-1}(\boldsymbol{P}) = \boldsymbol{s}^\top \boldsymbol{P}}$$

- **controlled multiplication**
- **makes LWE fail**
  (no protection for $\boldsymbol{s}$)

$\boldsymbol{s}^\top \boldsymbol{B} \cdot \boldsymbol{B}^{-1}(\boldsymbol{0})$ small
$\$ \cdot \boldsymbol{B}^{-1}(\boldsymbol{0})$ random

Evasive LWE. (conditional protection for $\boldsymbol{s}$)

*“The **only meaningful way** to use $\boldsymbol{B}^{-1}(\boldsymbol{P})$ is to **multiply it to** $\boldsymbol{s}^\top \boldsymbol{B}$ and ignore noise correlation.”*

$$\text{if} \quad (\boldsymbol{B}, \boldsymbol{P},\ \boldsymbol{s}^\top \boldsymbol{B} + \boldsymbol{e}_{\mathrm{B}}^\top,\ \boldsymbol{s}^\top \boldsymbol{P} + \boldsymbol{e}_{\mathrm{P}}^\top) \approx (\boldsymbol{B}, \boldsymbol{P},\ \$,\ \$)$$

$$\text{then} \quad (\boldsymbol{B}, \boldsymbol{P},\ \boldsymbol{s}^\top \boldsymbol{B} + \boldsymbol{e}_{\mathrm{B}}^\top,\ \boldsymbol{B}^{-1}(\boldsymbol{P})\ ) \approx (\boldsymbol{B}, \boldsymbol{P},\ \$,\ \boldsymbol{B}^{-1}(\boldsymbol{P}))$$

Evasive LWE and Evasive IPFE [HLL24]

IPFE (full protection) ⇐

Corporate needs you to find the differences between this picture and this picture.
They're the same picture.

Pairing. controlled multiplication

DDH or GGM. some protection

Lattice Trapdoor. controlled multiplication

LWE + Evasive LWE. some protection

⇒ Evasive IPFE

suffices for ABE *

Functionality.

$$\mathrm{Dec}(\mathrm{impk}, \boldsymbol{v}, \mathrm{isk}(\boldsymbol{v}), \mathrm{ict}(\boldsymbol{u})) = \boldsymbol{u}^\top \boldsymbol{v} + e_{\mathrm{Dec}}$$

Security.

$I \times J$ fresh noises

if $\{\boldsymbol{u}_i^\top \boldsymbol{v}_j + e_{ij}\}_{ij} \approx \{\$\}_{ij}$

then

$$(\mathrm{impk}, \{\boldsymbol{v}_j, \mathrm{isk}(\boldsymbol{v}_j)\}_j, \{\mathrm{ict}(\boldsymbol{u}_i)\}_i)$$
$$\approx (\mathrm{impk}, \{\boldsymbol{v}_j, \mathrm{isk}(\boldsymbol{v}_j)\}_j, \{\mathrm{ict}(\$)\}_i)$$

* not the full story, but good enough for now

Achievements of [HLL24]

Lattice-Based CP-ABE. for circuits (using garbling from [BGGHNSVV14])

- from LWE + evasive LWE
- **Previous.** [BV20] no security proof
  [W22] from LWE + tensor LWE + evasive LWE

ABE for Uniform. for DFA, L (using garbling from [LL20a])

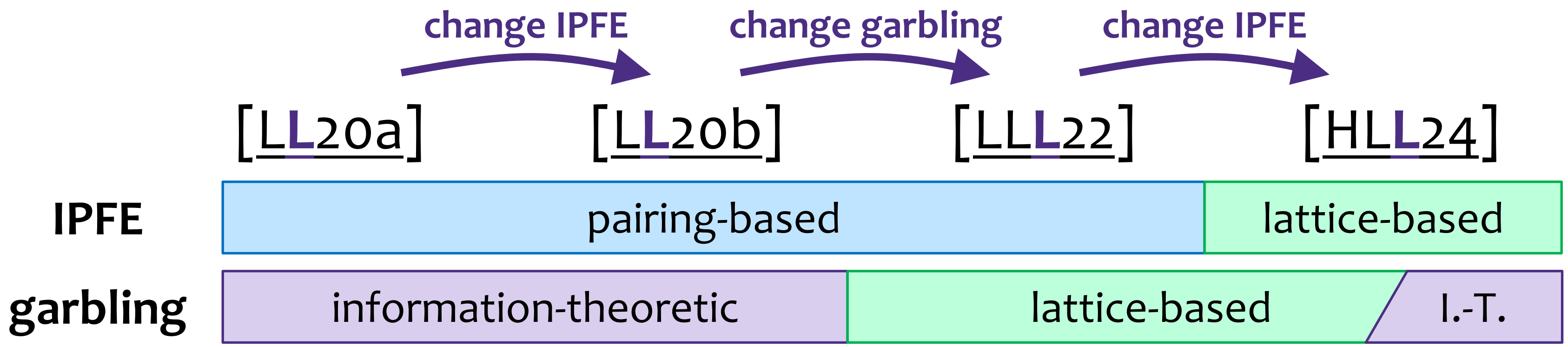
- **first** lattice-based public-key ABE for uniform
- **Previous.** [AS17,W22] against bounded collusion
  [AMY19] secret-key ABE for NFA
  [W22] no security proof

Summary of Paradigm

ABE $\Leftarrow$ IPFE $\circ$ Garbling

Composition of Security.

- IPFE – only rerandomized garblings revealed
- assumption – garblings are properly rerandomized
- garbling – secret/message hidden if unauthorized



change IPFE

change garbling

change IPFE

[LL20a] [LL20b] [LLL22] [HLL24]

IPFE: pairing-based | lattice-based

garbling: information-theoretic | lattice-based | I.-T.

Modular. **Hides** most ***raw*** usage of computational assumptions into IPFE and garbling security.

Powerful. Achieves various ABE with **better** properties.

Versatile. Works with pairing, lattice, or pairing + lattice.

Open Questions from Part I

- Gap between **selective/adaptive** ABE from **static** pairing assumptions

  (arithmetic span program vs ABP)

- **CP**-ABE for circuits from **falsifiable** lattice assumptions
- ABE for **DFA** from **falsifiable** lattice assumptions

  (Evasive LWE is non-falsifiable.)

Part II. **More**

Nothing technical now,
just the results and the messages.

Bounded and Unbounded (KP-ABE)

Recall. $\boldsymbol{s}^\top(\boldsymbol{A} - \boldsymbol{x} \otimes \boldsymbol{G}) + \boldsymbol{e}^\top \xrightarrow{\text{EvalCX}} \boldsymbol{s}^\top(\boldsymbol{A}_C - C(\boldsymbol{x}) \cdot \boldsymbol{G}) + \boldsymbol{e}_C^\top$

noise growth **exponential** in depth d of C

computation in $\mathbb{Z}_q$ – only works when $d = O(\log q)$

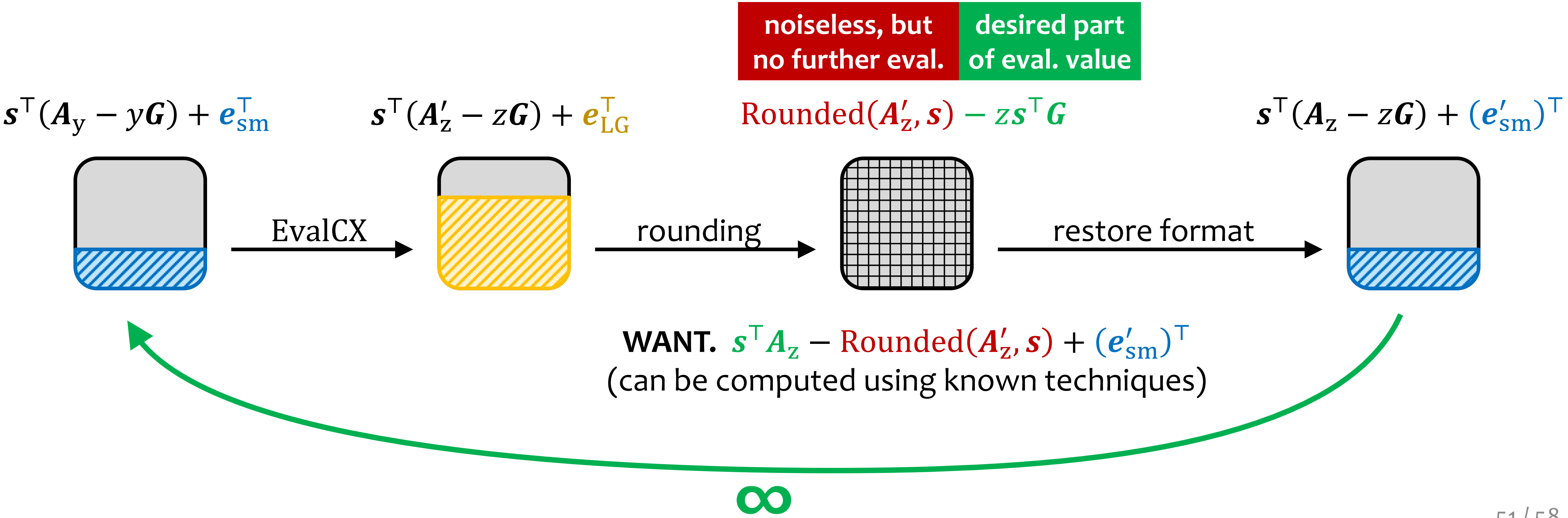
- q is often chosen upon Setup.
- q **must** be chosen upon Enc.
  - $\text{ct}_{\boldsymbol{x}}$ contains elements in $\mathbb{Z}_q$ (attribute encoding).
  - This forces $d \le |\text{ct}|$,
    so $\text{ct}_{\boldsymbol{x}}$ **cannot** work with sk_C if $d > |\text{ct}|$,
    even if $C(\boldsymbol{x}) = 0$.
  - "$\text{ct}_{\boldsymbol{x}}$ places an upper bound on d," (**depth-bounded**)
    even though $\boldsymbol{x}$ has nothing to do with d.

WANT. No upper bound on d from mpk, ct ("**depth-unbounded**").

Unbounded Evaluation for Attribute Encoding

Idea. (similar to *fully homomorphic encryption*)

- Start from ***somewhat small*** noise.
- Perform some evaluation. Noise becomes ***somewhat large***.
- **Reduce noise** to ***somewhat small*** before it ***overflows***.
- Rinse and repeat.



noiseless, but no further eval.

desired part of eval. value

$\boldsymbol{s}^\top(\boldsymbol{A}_\mathrm{y} - y\boldsymbol{G}) + \boldsymbol{e}_\mathrm{sm}^\top$

EvalCX

$\boldsymbol{s}^\top(\boldsymbol{A}'_\mathrm{z} - z\boldsymbol{G}) + \boldsymbol{e}_\mathrm{LG}^\top$

rounding

$\mathrm{Rounded}(\boldsymbol{A}'_\mathrm{z}, \boldsymbol{s}) - z\boldsymbol{s}^\top\boldsymbol{G}$

restore format

$\boldsymbol{s}^\top(\boldsymbol{A}_\mathrm{z} - z\boldsymbol{G}) + (\boldsymbol{e}'_\mathrm{sm})^\top$

WANT. $\boldsymbol{s}^\top\boldsymbol{A}_\mathrm{z} - \mathrm{Rounded}(\boldsymbol{A}'_\mathrm{z}, \boldsymbol{s}) + (\boldsymbol{e}'_\mathrm{sm})^\top$

(can be computed using known techniques)

∞

Achievements and Open Questions [HLL23]

KP-ABE for circuits of unbounded depth.

- long-standing open problem
- from circular LWE + evasive circular LWE (circular = encrypt $\boldsymbol{s}$ using $\boldsymbol{s}$)

- **Previous.** [BGGHNSVV14] for bounded depth
- **Concurrent.** [CW23] for bounded depth with $|\mathrm{sk}_C| = O(1)$

Related Primitives. (with depth bound removed, from circular LWE)

Open Questions. depth-unbounded KP-ABE
from **falsifiable** (no “evasive”) lattice assumptions

Dream and Actual Versions of ABE [JLL23]

Previous. ABE for ABP, NL, circuits…

ABE for RAM (best model for real-world programs)

$$|\mathrm{sk}_f| = O(1)$$

$$|\mathrm{ct}_x| = O(1)$$

$$T_{\mathrm{Dec}} = O(T_{\mathrm{RAM},f,x})$$

possible that $T_{\mathrm{RAM},f,x} < |x|$ (think binary search)

from functional encryption for circuits
with *arbitrarily bad* efficiency… ↓

$\lvert\mathrm{sk}_f\rvert$	$\lvert\mathrm{ct}_x\rvert$	T_{Dec}
$O(1)$	$O(1)$	$O(T + \lvert f\rvert + \lvert x\rvert)$
$\lvert f\rvert + O(1)$	$O(1)$	$O(T + \lvert x\rvert)$
$O(1)$	$\lvert x\rvert + O(1)$	$O(T + \lvert f\rvert)$
$\lvert f\rvert + O(1)$	$\lvert x\rvert + O(1)$	$O(T)$

Are we (am I) stupid,
or is it some necessary evil?

YOU CAN (NOT) OPTIMIZE [L24]

Theorem. For any secure ABE supporting $P(f = i,\ \ x = \boldsymbol{R}) = \boldsymbol{R}[i]$, it holds that

$$|\mathrm{ct}_x| \cdot T_{\mathrm{Dec}} = \Omega(|x|).$$

Similar trade-off lower bound holds between $|\mathrm{sk}_f|$ and T_{Dec}.

$\Longrightarrow$ Schemes of [JLL23] are **Pareto-optimal.**

Fact. Schemes of [JLL23] can be modified into

$$|\mathrm{sk}_f| = |f|^{\alpha} + O(1), \quad |\mathrm{ct}_x| = |x|^{\beta} + O(1),$$
$$T_{\mathrm{Dec}} = O\big(T \times (|f|^{1-\alpha} + |x|^{1-\beta})\big),$$

for any constants $0 < \alpha, \beta < 1$.

$\Longrightarrow$ The trade-off lower bound is **tight** for $T = O(1)$.

Achievements of [JLL23,L24]

New Agenda.

- multi-objective optimization
- quest for Pareto-optimality

Trade-Off Lower Bounds for ABE.

- first such bounds
- **Message.** Maybe succinctness is not worth it
  if we must pay dearly for each decryption?

Constructions.

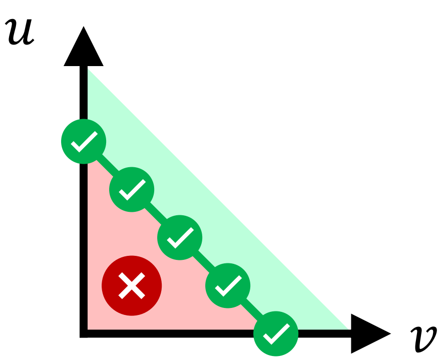
- down-to-constant optimization
- Pareto-optimal

Open Questions from [JLL23,L24]

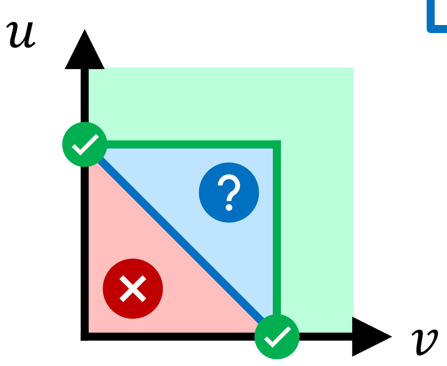
- fully pin down the Pareto frontier for general T

$|\mathsf{ct}_x| = O(|x|^u)$
$T_{\mathsf{Dec}} = O(T + |x|^v + \cdots)$

($T = O(1)$)



(general T)



- Is "f, x verbatim for free" the correct cost model?

$\mathsf{Dec}(\mathsf{mpk}, \mathsf{sk}'_f, \mathsf{ct}'_x)$

from verbatim-for-free model. $\mathsf{ct}'_x = (x, \mathsf{ct}_x)$
cannot achieve $T_{\mathsf{Dec}} = O(T)$ with $|\mathsf{ct}'_x| = |x| + O(1)$.

other implementation achieving the goal? (ct'_x encoding x in some clever way)

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