How to Simulate Random Oracles with Auxiliary Input

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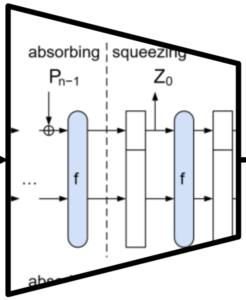
Random Oracle Model [BR93]

Standard-Model (Real-World) Hash Functions

$\{0,1\}^n$ or $\{0,1\}^*$ →



→ $\{0,1\}^\lambda$

✗ no control over how code is used
⟹ difficulty in analyzing security

Random Oracle Model

x ←

$\mathcal{O}(x)$ →

$\mathcal{O}: \{0,1\}^{n/*} \to \{0,1\}^\lambda$
random truth table

✓ easier security analysis
- **practical schemes**
- **qualitative** – what property?
- **quantitative** – (T, ε)-security

✗ overly optimistic
against non-uniform adversaries

Non-Uniformity, Examples of Over-Optimism

Non-Uniform Adversary = machine A, sequence $\{z_\lambda\}_{\lambda\in\mathbb{N}}$ of strings (advice)
on security parameter λ: $A(1^\lambda, z_\lambda) \leftrightarrow$ scheme
potential benefits of reusable preprocessing (e.g., rainbow tables [O03])

⚠ Advice depends on H.
ROM does **not** capture $\mathcal{O}$-dependent advice.

Qualitative

ROM. $\mathcal{O}$ is a keyless collision-resistant (CR) hash function.

Fact. Keyless functions cannot be CR against non-uniform adversaries.
(advice = smallest collision)

Quantitative

ROM. to invert $y = \mathcal{O}(x)$ w.p. $\Omega(1)$ given y, need $Q = \Omega(2^\lambda)$ queries.

(heuristic) $\Longrightarrow$

to invert $y = H(x)$ for "good" H, need $T = \Omega(2^\lambda)$.

Fact. to invert $y = H(x)$, (S = advice length) $S, T = O(2^{3\lambda/4})$ suffices [FN91].

Auxiliary-Input Random Oracle Model [U07]

adversary = machine A, function ai

non-computable ai allowed; output length S depends on λ.

on security parameter λ:

- $z \leftarrow \mathrm{ai}(\lambda, \mathcal{O})$
- $A(1^\lambda, z) \leftrightarrow \mathcal{O}$ and scheme

✓ **avoids overly optimistic heuristics**

- **qualitative** – no seedless CRHF (advice = collision, depends on $\mathcal{O}$)
- **quantitative** – [FN91] attack carries over to AI-ROM

✗ **difficulty in analyzing security**

No clue even for proving DDH in AI-ROM from DDH in non-uniform standard model!

- ROM – can lazy-sample/program truth table
- AI-ROM – $A(1^\lambda, z) \leftrightarrow \mathcal{O}$ how to handle **arbitrary correlation**?

Primary Goal, Previous/Our Results

"Develop methods for showing (tight) security in AI-ROM."

Known Result. Presampling method [U07,CDGS17]
to "simulate" AI-ROM information-theoretically,
as tight as it can be, but not tight.

Our Main Result. A new technique
to simulate AI-ROM computationally,
much tighter!

tight. $T_{\text{reduction}} \sim T_{\text{underlying}}$; qualitatively – **same/stronger** security from **weaker/same** assumptions.

Why Two Colors for Simulation?

open since AI-ROM conception [U07]

Two-Stage Definition (Distinguisher-Independent Simulation)

1. let $A(z) \leftrightarrow \mathcal{O}$ and record transcript τ of A
2. $D(\tau) \rightarrow \{0,1\}$ (no $\mathcal{O}$!)

Requirement is $\forall(A, \mathrm{ai})\ \exists(S, \mathrm{nu})\ \forall D\ \ldots$

- "nu" for (standard-model) **n**on-**u**niformity
- **zero-knowledge** uses **two-stage** definition

Simulated Step 1.

- $(z, w) \leftarrow \mathrm{nu}(\lambda)$
- let $A(z) \leftrightarrow S(w)$

***w* = extra advice**

Single-Stage Definition (Distinguisher-Dependent Simulation)

- let $A(z) \leftrightarrow \mathcal{O}$ and A outputs a bit

Requirement is $\forall(A, \mathrm{ai})\ \exists(S, \mathrm{nu})\ \ldots$

(think of D merged inside A)

Comparison of Simulation Methods

method / assumption	extra advice	time per query
presampling [U07,CDGS18]	$\Theta(\lambda n S\, Q/\varepsilon)$	$\Theta(\lambda n S\, Q/\varepsilon)$ [circuit] or $\tilde{O}(n)$ [RAM]
subexp.sec. PRF	$\mathrm{poly}(\lambda, n, S)$	$\mathrm{poly}(\lambda, n, S)$
exp.sec. quasi.lin.time PRG	$O(\lambda + S)$	$n \cdot \tilde{O}(\lambda + S)$
faster-than-secure PRF	$O(n^{1+\gamma}(\lambda + S)^{1+\gamma})$	$n^{1+\gamma} \cdot \mathrm{polylog}(\lambda, n, S)$ [RAM]

$\gamma > 0$ arbitrarily tunable

- Showing $\mathcal{O}: \{0,1\}^n \to \{0,1\}^\lambda$.
- Presampling requires knowing Q, ε in advance
  - (due to ε,) **fails** two-stage definitions (e.g., zero-knowledge).
    - **^ would need super-poly.-time sim. for negl. ε**
- **All ours satisfy two-stage definition.**
- S, Q are (adversarial resources) large $\mathrm{poly}(\lambda)$ [even $2^{\Theta(\lambda)}$ in concrete security].
  - **Optimization Priority.** $S, T, Q, \varepsilon \succ n \succ \lambda$ – **Last 2 of ours are tighter.**

Key Idea

✓ **first NIZK in AI-ROM**

Q. The simulator is **efficient** and **looks like a random function.** What can it be?

A. A pseudorandom function (PRF).

✓ **resolves open problem of AI-ROM simulation** (two-stage definition)

even better – does not depend on A

Let's try it… [$\langle \cdots \rangle$ = transcript]

$$\langle A(\mathrm{ai}(\mathcal{O})) \leftrightarrow \mathcal{O} \rangle \overset{?}{\approx} \langle A(\mathrm{ai}(F(k,\cdot))) \leftrightarrow F(k,\cdot) \rangle$$

✗ ai: arbitrarily complex

$$\overset{(1)}{\approx} \langle A(\mathrm{ai}'(\mathcal{O})) \leftrightarrow \mathcal{O} \rangle$$

$$\overset{(2)}{\approx} \langle A(\mathrm{ai}'(F(k,\cdot))) \leftrightarrow F(k,\cdot) \rangle$$

Fix (leakage simulation lemma [CCL18]). Let

$$X = \mathcal{O}, \quad Z = \mathrm{ai}(X) \text{ (of length } S\text{)},$$

then $\exists \mathrm{ai}'$ of complexity $\sim 2^S ST/\varepsilon^2$:

$$(X, Z) \approx_{T,\varepsilon} (X, \mathrm{ai}'(X)).$$

"function of output length S, complexity controlled at $\sim 2^S$"

(1) set $T = 2^\lambda > T_A + T_D$ and $\varepsilon = 2^{-\lambda}$

(2) tune up λ_{prf} for security against total time ($\sim 2^{S+\lambda}$)

requires subexp.-secure PRF

Tight Reduction = Fast and Secure PRF

Convention. at λ_{prf}, the PRF is ε-secure in $2^{\lambda_{\mathrm{prf}}}$ time.

- fixed level of security
- compete for small T_{prf} (plus, short k = extra advice)

$T_{\mathrm{prf}} \in \mathrm{subpoly}(\lambda)$
"faster than secure"

Typical. $T_{\mathrm{prf}} = n \times \Omega(\lambda_{\mathrm{prf}})$

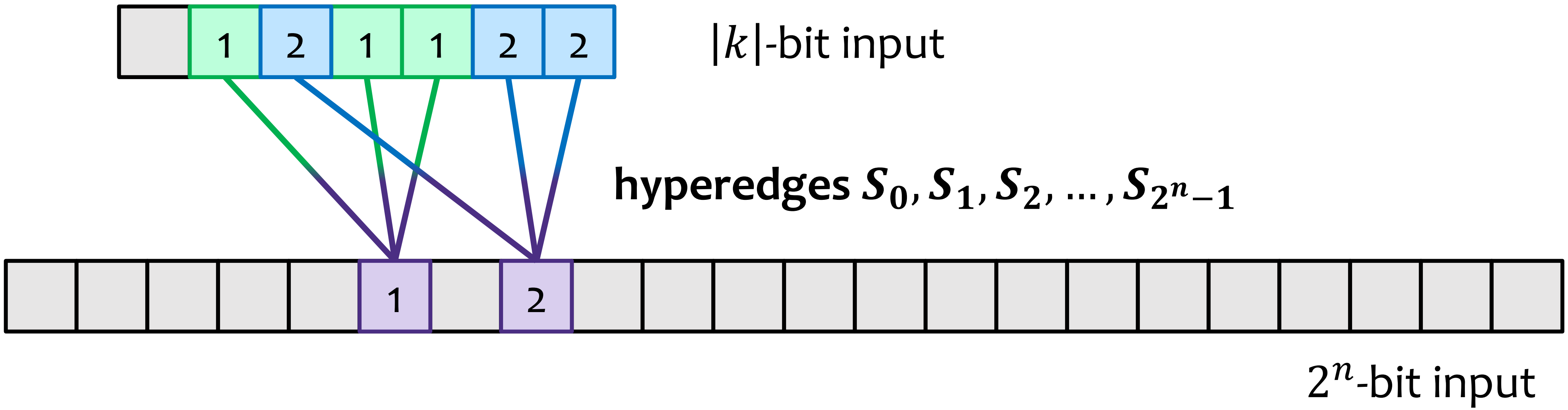
- $\lambda_{\mathrm{prf}} \geq S + \lambda$

adversary-dependent degrades quickly

Wish. What about $T_{\mathrm{prf}} = n \cdot \mathrm{polylog}(\lambda_{\mathrm{prf}})$?

[AR16] **weak** PRF with $T_{\mathrm{wprf}} = \tilde{O}(n)$ from Goldreich's PRG [Goo]

Faster-than-Secure PRF from Goldreich's PRG



$F(k, x) = P(k[S_x])$ with fixed function P [e.g., XOR-MAJ]

- **cryptanalysis.** exp.-PRF if $(S_x)_x$ is sufficiently expanding [e.g., $(t, 0.99)$ for $t \sim \lambda_{\text{prf}}^{1+\gamma}$]

✓ locality = P's input length = $\Theta(n)$

? represent exp.-size hypergraph succinctly (extra advice)

? compute S_x efficiently (time per query)

Representing and Accessing Expanders

2^n-size hypergraph must be $(t, 0.99)$-expanding, $t \sim \lambda_{\mathrm{prf}}^{1+\gamma}$.

- $S_x = h(x)$ is expanding w.h.p. with t-wise independent h
- let h be a degree-t polynomial

Evaluate $x \mapsto h(x) \mapsto S_x$ in time $n \cdot \mathrm{o}(\lambda_{\mathrm{prf}})$?

- fast polynomial evaluation [KU09]
- coefficients $\Longrightarrow$ preprocess $\Longrightarrow$ size $t^{1+\gamma} \log^{1+\gamma} p$ (p for field)
- **online.** evaluation time is $(\log^{1+\gamma} p) \cdot \mathrm{polylog}(t) \sim n^{1+\gamma}\, \mathrm{polylog}(\lambda_{\mathrm{prf}})$

first NIZK in AI-ROM

$\Uparrow$

Leakage Simulation + Subexponentially Secure PRF $\Longrightarrow$ AI-ROM simulation

"Faster-than-Secure" $\Longrightarrow$ very tight

Thanks!

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another trick: combine presampling + ours
for best of both worlds [see paper!]