

# The Pseudorandom Oracle Model and Ideal Obfuscation

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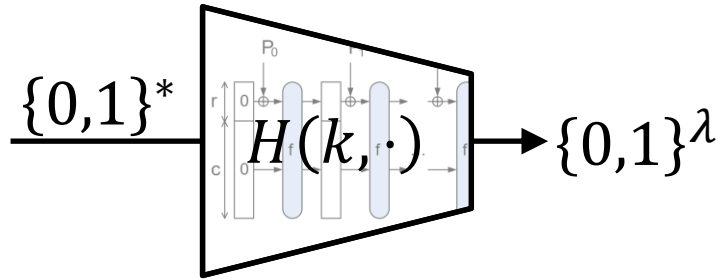
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 **NTTResearch**

# Outline

- Motivation and Result
- **Pseudorandom Oracle Model (PrOM)**
- Ideal Obfuscation in PrOM

# Hash Functions



## basic, useful primitive

- one-wayness
- second-preimage resistance
- collision resistance

$$\text{Sign}(sk, m) \stackrel{\text{def}}{=} \text{Sign}_\lambda(sk_\lambda, H(k, m))$$

✓ collision resistance

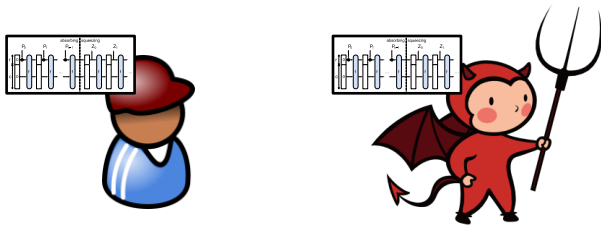
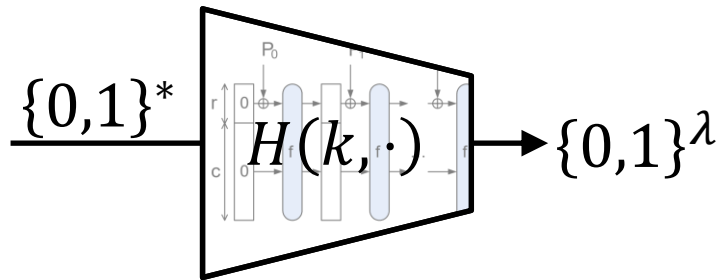
$$\text{Sign}(sk, m) \stackrel{\text{def}}{=} \text{TDP}^{-1}(sk, H(k, m))$$

🤔 What assumption for  $H$ ?

**desired security not always captured by simple complexity assumptions**

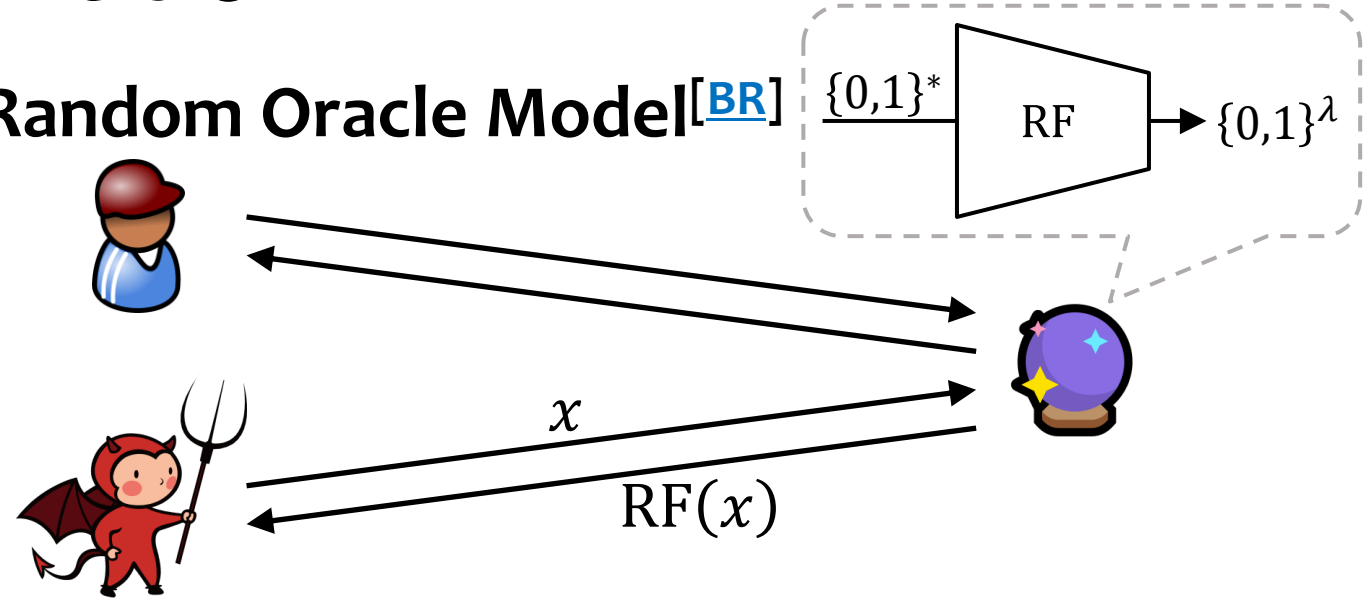
# Hash Functions: Idealization

## Standard Model



- one-wayness
- second-preimage resistance
- collision resistance
  
- correlation intractability

## Random Oracle Model [BR]



- ✓ **practically used** and **more efficient** schemes  
Schnorr signature [Sch], RSA-OAEP [BR], TLS [KPW,DFGS,DJ]...
- ⚠ (contrived) **uninstantiability** results [CGH]
- ✓ proof in ROM >> no proof  
▶ if **cannot** write proof in ROM
- ✓ **precursor** to standard-model version

# Obfuscation (for Circuits)

```
basicFun = Function[{Typed[pixel0, "ComplexReal64"]},  
  Module[{iters = 1, maxiters = 1000, pixel = pixel0},  
    While[iters < maxiters && Abs[pixel] < 2,  
      pixel = pixel^2 + pixel0;  
      iters++  
    ];  
    iters];
```

Obf(·)

```
1 #include <stdio.h> 29  
2 #include <malloc.h> 30  
3 #define ext(a) (exit(a),0) 31  
4 #define I "...;+<?F7RQ&#+" 32  
5 #define a "%s\n" 33  
6 #define n "0?\n" 34  
7 #define C double 35  
8 #define o char 36  
9 #define l long 37  
10 #define L sscanf 38  
11 #define i stderr 39  
12 #define e stdout 40  
13 #define r ext (1) 41  
14 #define s(O,B) L(++j,O,&B)!18&c++q&&L(v[q],O,&B)!18&--q 42  
15 #define F(U,S,C,A) t=0,++j&&(t=L(j,U,&C,&A)),(t&&c++q&&!t=L(v[q],U, 43  
16 #define T(E) (s("%d",E),E)|(fputs(n,i),r)) 44  
17 #define d(C,c) (F("%lg,%lg", "%lg", C,c)) 45  
18 #define O (F("%d,%d", "%d", N,U), (N&&U)|(fputs(n,i),r)) 46  
19 #define D (s("%lg", f)) 47  
20 #define E putc 48  
21 49  
22 C 50  
23 G=0, 51  
24 R 52  
25 =0,Q,H 53  
26 ,M,P,z,S 54  
27 =0,x=0 55  
28 , f=0;l b,j=0, k 56
```

$$\text{Obf}(C)(x) = C(x)$$

Obf(C) is  
“unintelligible”

# Indistinguishability Obfuscation

$$C_0 \equiv C_1, |C_0| = |C_1| \implies \text{Obf}(C_0) = \text{Obf}(C_1)$$

✓ already very **powerful**

deniable encryption, short signatures, injective TDF, NIZK, OT<sup>[SW]</sup>

FHE<sup>[ABFGGTW]</sup>      FE for P<sup>[GGHRSW]</sup>      2-round MPC<sup>[GGHR]</sup>

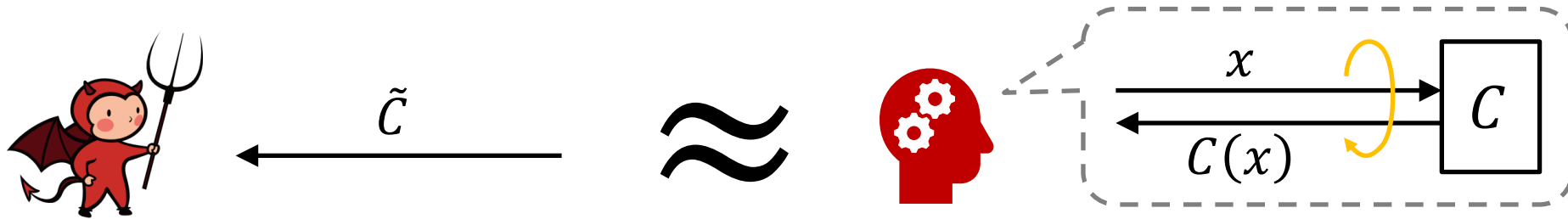
succinct garbled RAM<sup>[CH]</sup>      (+ many more!)

✓ from **well-studied assumptions**<sup>[JLS]</sup>

⚠ **weak, unintuitive** security guarantee

⚠ **complicated design** in applications

# Simulation-Secure Obfuscation (Idealized)



## Ideal Obfuscation.

$$\exists S \quad \forall C:$$

$$\text{Obf}(C) \approx S^C(1^{|C|}, 1^{|x|})$$

**X impossible** for **unlearnable** circuits

## Virtual Black-Box.

$$\forall \text{1-bit } A \quad \exists S_A \quad \forall C:$$

$$A(\text{Obf}(C)) \approx S_A^C(1^{|C|}, 1^{|x|})$$

**⚠ VBB not possible in general** [\[BGIRSVY\]](#)  
(contrived “self-eating” programs)

✓ **strong, intuitive** security guarantee

✓ **simple, intuitive** designs in applications

✓ **only path to certain (plausible) applications**

doubly efficient PIR [\[BIPW\]](#) FHE for RAM [\[HHWW\]](#)

VGB obfuscation [\[BC\]](#) time-optimal FE for RAM [\[ACFQ\]](#)

OT from binary erasure channel [\[AIKNPPR\]](#)

public-coin  $d\mathcal{O}$ , unbounded-input obfuscation for TM [\[IPS\]](#)

input-hiding obfuscation for evasive functions [\[BBCKPS\]](#)

extractable WE [\[GKPVZ\]](#) wiretap-channel coding [\[IKLS\]](#)

refuting dream XOR lemma [\[BIKSW\]](#)

# Motivation

CRHF  $\xrightarrow{\text{idealize}}$



**random oracle**

Can non-black-box use of hash function help?

$$RO = \text{Obf}(\text{PRF}(k, \cdot))$$



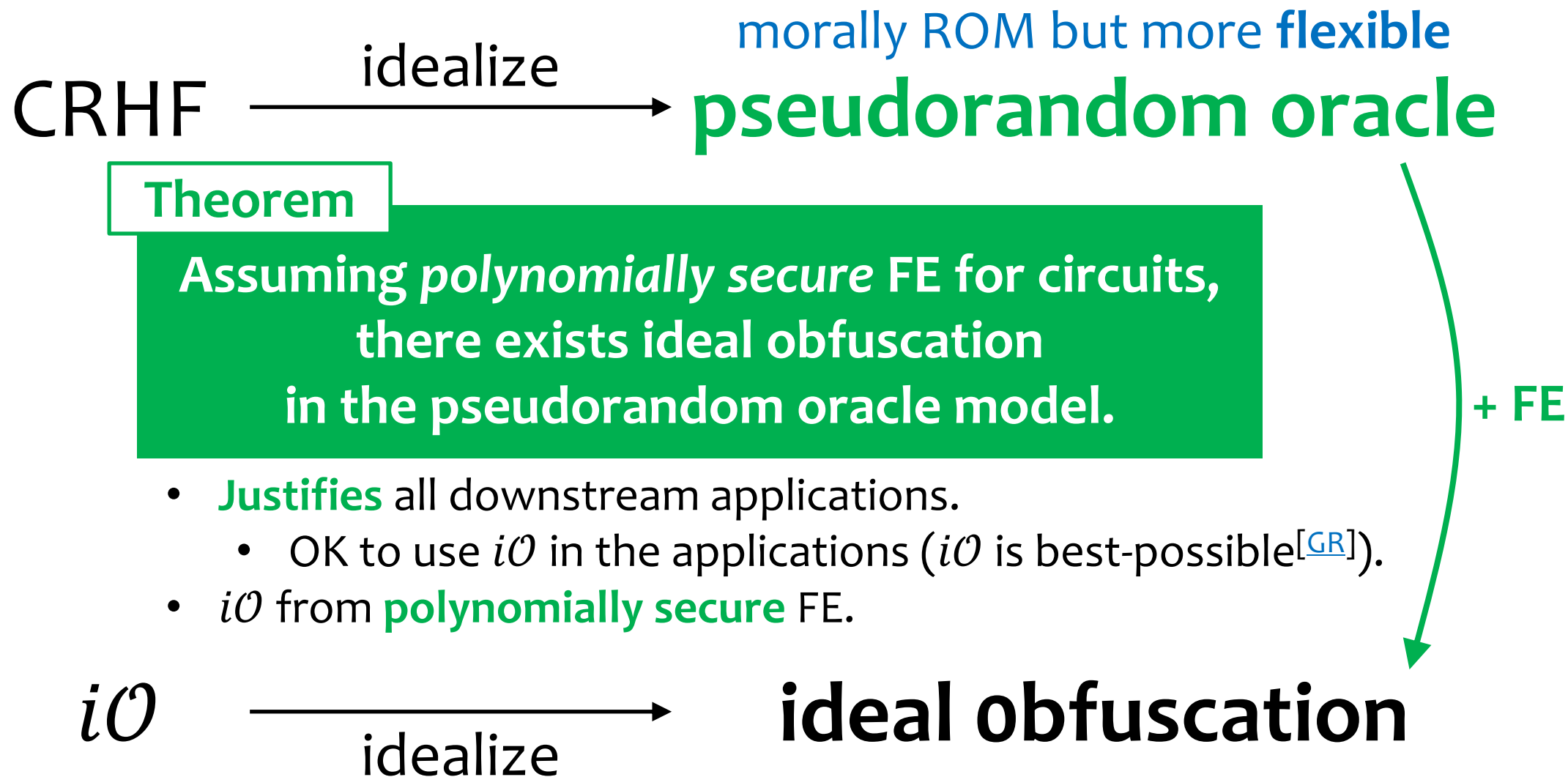
**Black-box** use of hash function does **not** help building ideal obfuscation. [\[CKP\]](#)

$i\mathcal{O}$   $\xrightarrow{\text{idealize}}$

**ideal obfuscation**



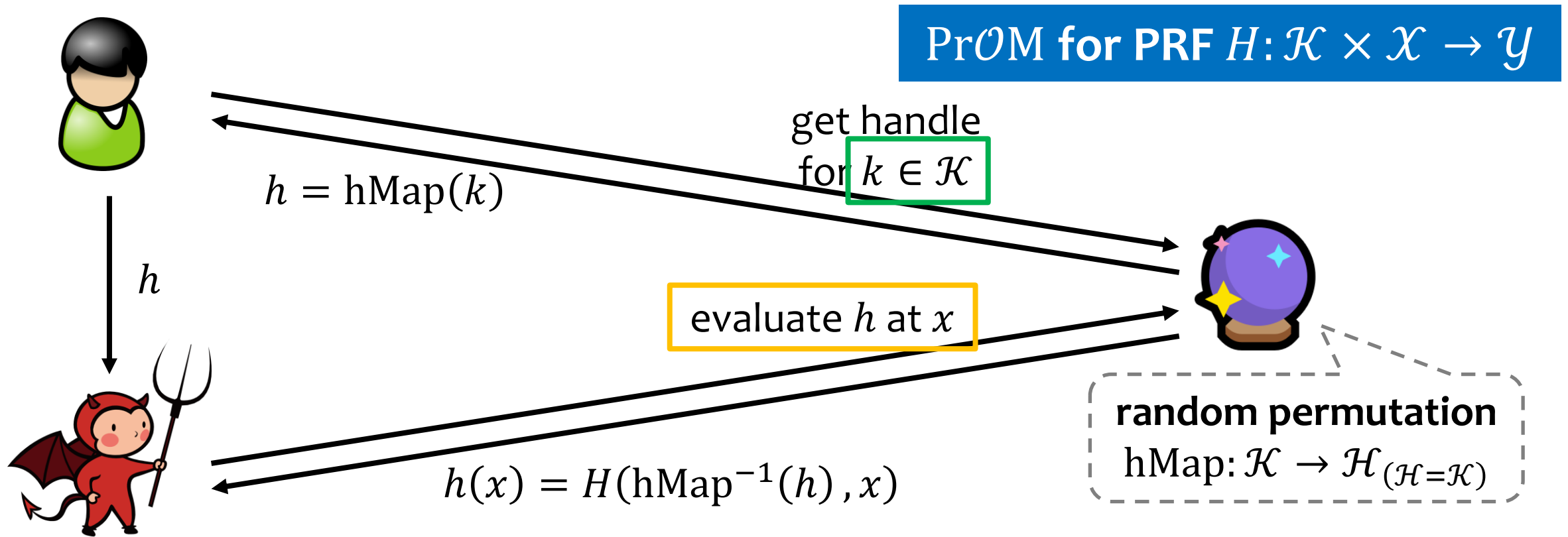
# Our Result



# Pseudorandom Oracle Model (PrOM)

Two aspects of the model.

- $H$  looks like a **random function** (using  $h$  is like ROM)
- $H$  has (short) **code** (using  $k$  is like usual PRF)



# Two Ways of Using $H$ in $\text{PrOM}$

- with  $h$  – call oracle to evaluate  $H$       **only use  $h$ : just ROM**
- with  $k$  – use code of  $H$       **only use  $k$ : just PRF**

**flexibility of  $\text{PrOM}$ : use in both ways, simultaneously**

But when  $k$  is seen by adversary,

$h$  is just a usual function, **not** a random oracle!

# Basic Recipe of Using PrOM



$h \leftarrow \text{hMap}(k \leftarrow \$), \text{ FHE/GC/FE}(k)$



$h(x) = H(k, x)$

$\Delta$   $h$  cannot be monitored or programmed



$h \leftarrow \text{hMap}(k \leftarrow \$), \text{ Sim}(\{H(k, x_i)\}_i)$

$h(x) = H(k, x)$

(FHE/GC/FE security)



$h, \text{ Sim}(\{\text{RF}(x_i)\}_i)$

$h(x) = \text{RF}(x)$

(PRF security of  $H$ )

$\checkmark$   $h$  can be monitored and programmed

# Limited Use of Code in PrOM

- with  $h$  – call oracle to evaluate  $H$       **only use  $h$ : just ROM**
- with  $k$  – use code of  $H$       **only use  $k$ : just PRF**

**flexibility of PrOM: use in both ways, simultaneously**

But when  $k$  is seen by adversary,

$h$  is just a usual function, **not** a random oracle!

**In hybrid with  $k$  removed,  $h$  becomes a random oracle!**

**limited use of code: must  $\approx$  hybrid without non-black-box use**

# The Random Oracle Paradigm<sup>[BR]</sup>

1. Formally define  $\Pi$  in the ROM.
  2. Design a scheme  $P^{\mathcal{O}}$  for  $\Pi$  in the ROM.
  3. Prove  $P^{\mathcal{O}}$  satisfies the definition for  $\Pi$  in the ROM.
  4. Instantiate  $\mathcal{O}$  by a real hash function.
- ✓ models intuition of hash functions being public random-looking functions
  - ✓ excludes generic attacks
  - ✓ heuristically justifies security of **practically used** / **more efficient** schemes
  - ⚠ forbids use of code of hash function
    - impossibility results**      simulation-secure FE<sup>[AKW]</sup>      VBB obfuscation<sup>[CKP]</sup>
    - ad hoc code use in ROM**      recursive proof composition<sup>[BCMS]</sup>
  - ⚠ (contrived) uninstantiability results

# The Pseudorandom Oracle Paradigm

1. Formally define  $\Pi$  in the **PrOM**.
  2. Design a scheme  $P^{\mathcal{O}}$  for  $\Pi$  in the **PrOM**.
  3. Prove  $P^{\mathcal{O}}$  satisfies the definition for  $\Pi$  in the **PrOM**.
  4. Instantiate  $\mathcal{O}$  by a real hash function.
- ✓ models intuition of hash functions being public random-looking functions **with code**
  - ✓ excludes generic attacks
  - ✓ heuristically justifies **goals beyond previous reach** (this work – **ideal obfuscation**)
  - ✓ allows limited use of code

⚠ (contrived) uninstantiability results

# How to Instantiate PrOM

good for ROM  $\implies$  good for PrOM

**Example.**  $h = k$  are random strings,  $H(k, x) = \text{SHA3}(k \parallel x)$

## Rationale.

- Good hash function is “**self-obfuscated PRF**”.
- In ROM instantiation, oracle calls are replaced by code of hash function anyway. Using it for PrOM **does not morally demand more** from this hash function **than for ROM**.



# Ideal Obfuscation: Definition

$\mathcal{O}$ : oracle of idealized model (e.g.,  $\text{Pr}\mathcal{O}^H$ )

$\text{Obf}^{\mathcal{O}}(C) \rightarrow \tilde{C}^{\bullet}$ : obfuscator

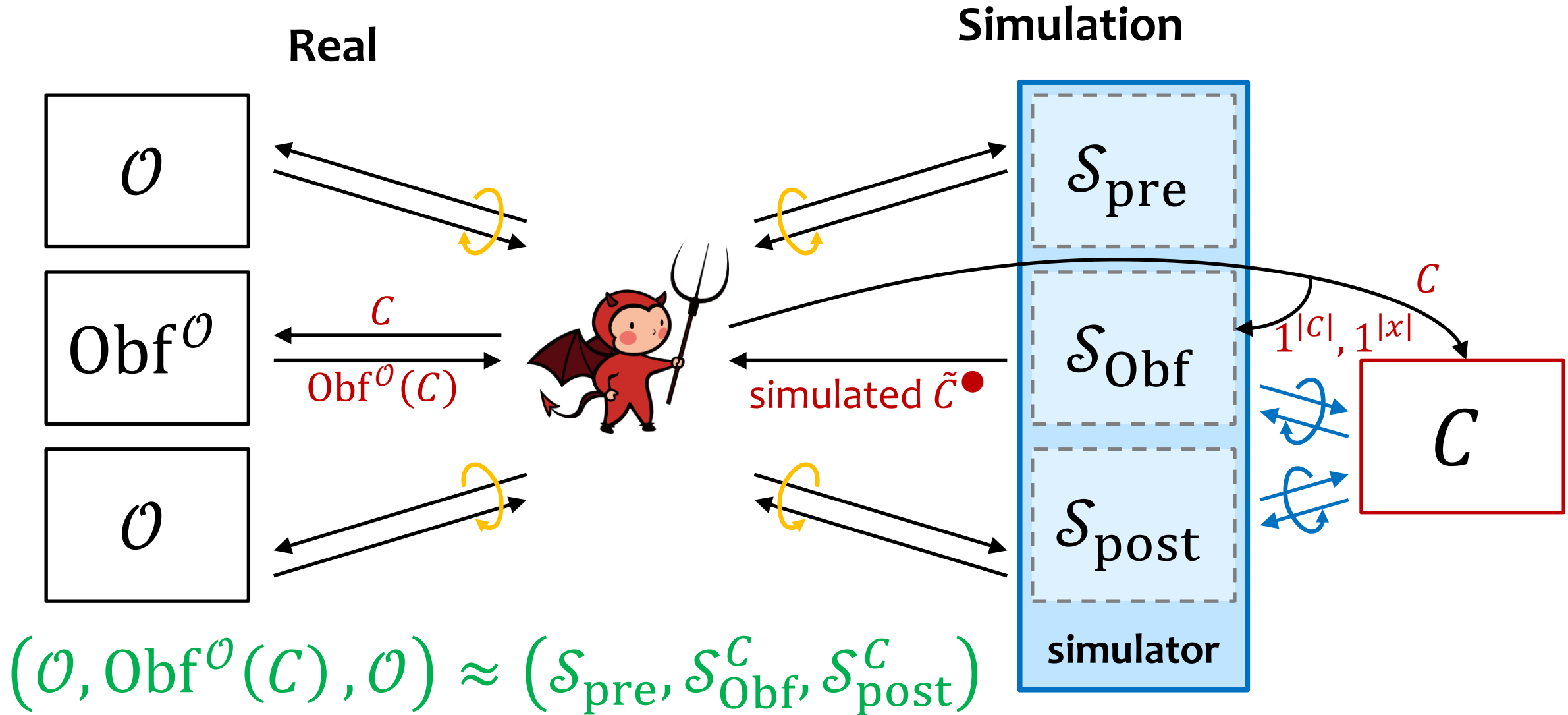
$C$  has **no oracle access**

$\tilde{C}^{\bullet}$  has **oracle access**

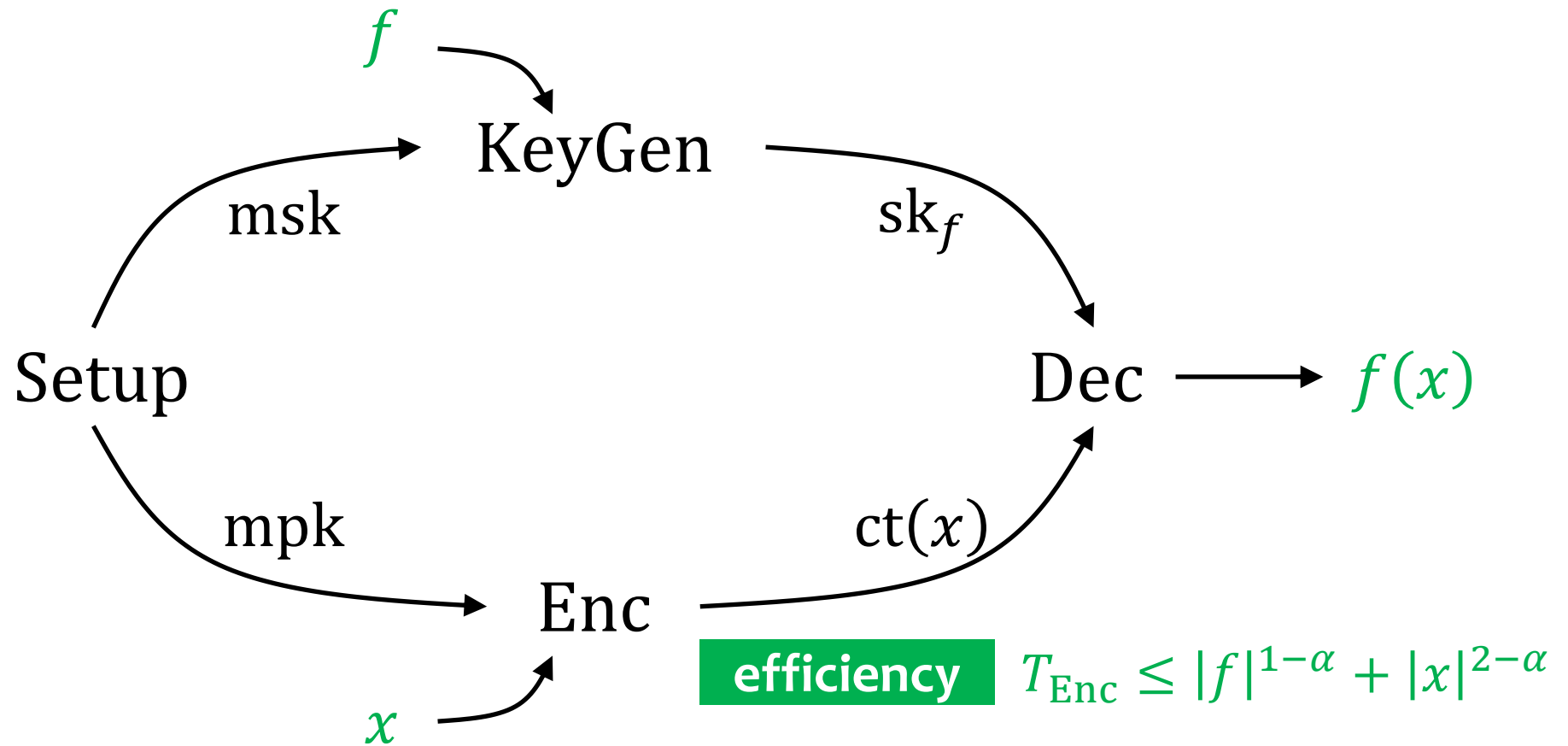
$$\forall C, x: \Pr[\tilde{C}^{\bullet} \leftarrow \text{Obf}^{\mathcal{O}}(C) : \tilde{C}^{\bullet}(x) = C(x)] = 1.$$

**Security** is indistinguishability-based. [\[MRH\]](#)

# Ideal Obfuscation: Security Definition



# Tool: (Standard-Model) Functional Encryption



**adaptive**  $x_0, x_1$  chosen after seeing  $mpk, sk_f$

**Security.**  $(mpk, sk_f, ct(x_0)) \approx (mpk, sk_f, ct(x_1))$  if  $f(x_0) = f(x_1)$

# Main Theorem

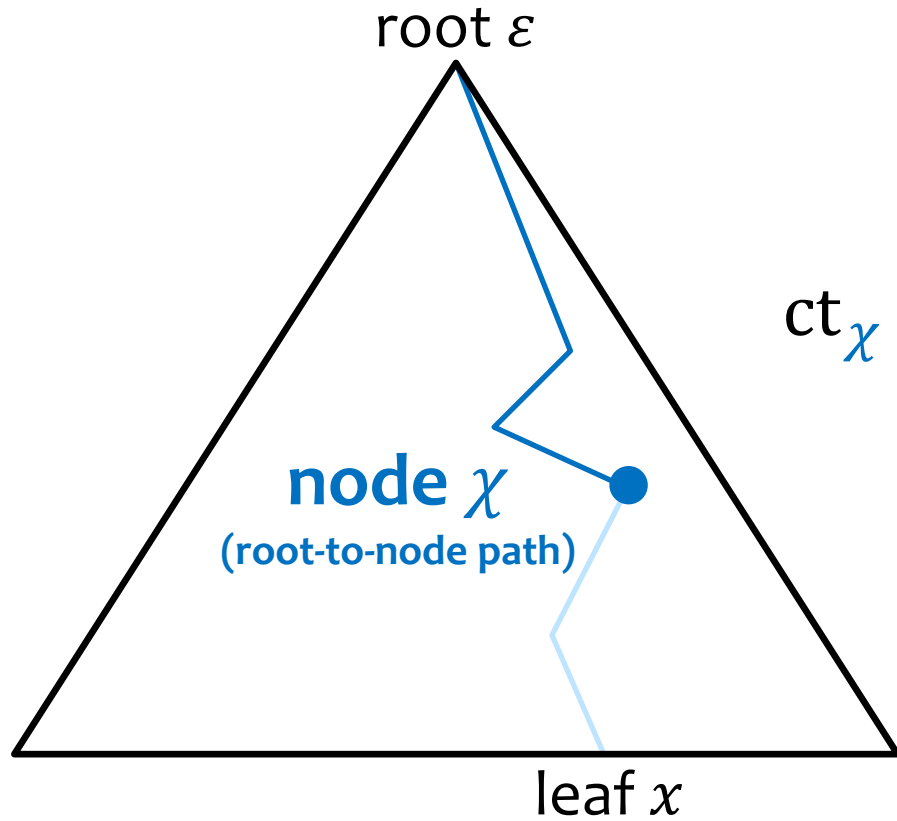
Assuming polynomially secure **succinct** FE for all circuits,  
there exists ideal obfuscation **for all circuits**  
in the pseudorandom oracle model **for any PRF  $H$** .

Function-revealing encryption ( $f$  in Setup, no KeyGen) is sufficient.

**Fact.** The FE/FRE we need can be based on polynomially secure selective 1-key sublinearly succinct public-key FE, thus on well-founded assumptions. [[ABSV](#), [GVW](#), [GS](#), [LM](#), [AJS](#), [BV](#), [AS](#), [KNTY](#), [JLL](#), [JLS](#)]

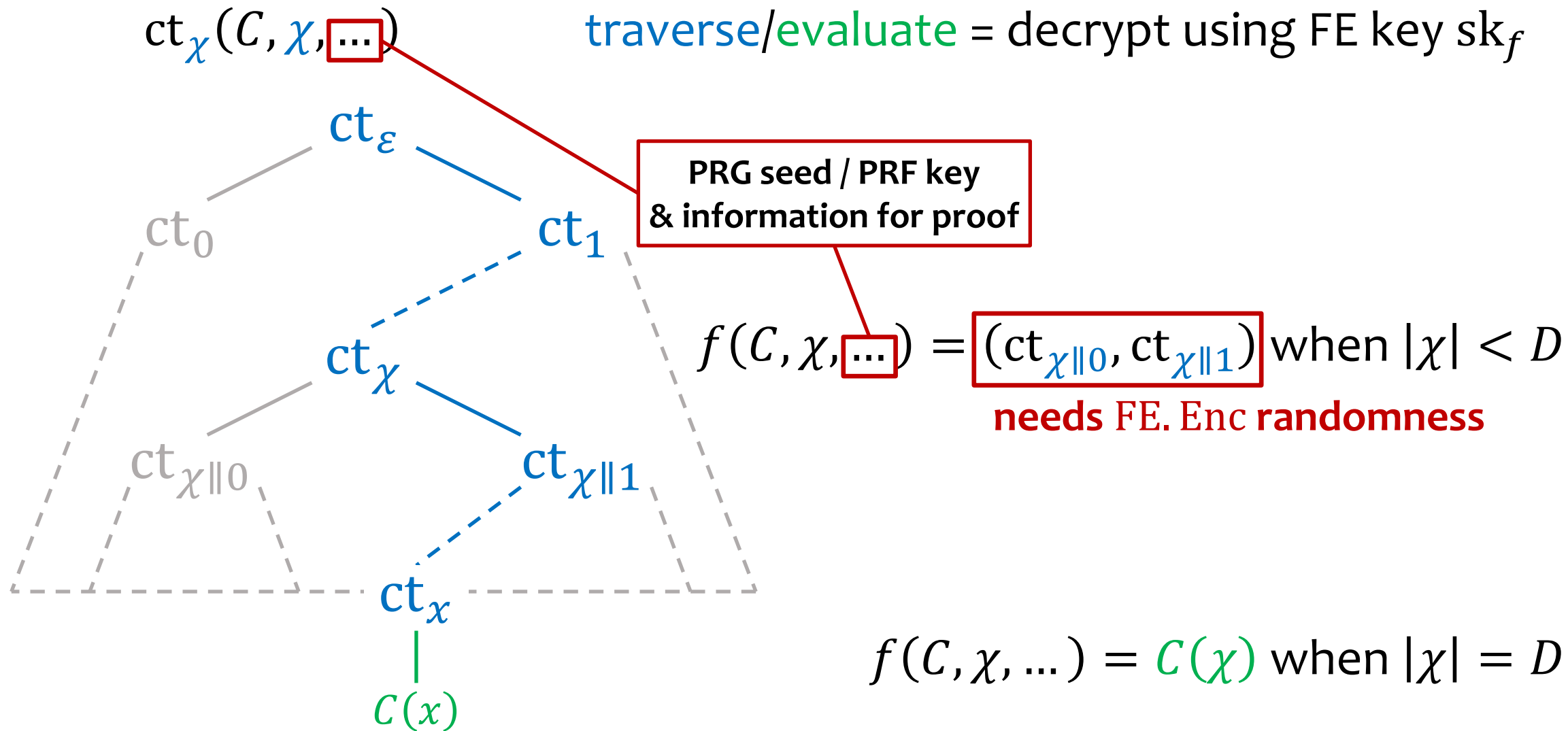
# $i\mathcal{O}$ from FE: Quick Recap

perfect binary tree (leaf  $\Leftrightarrow$  input)



$ct_\chi = \text{FE ciphertext of } (C, \chi, \dots) \text{ for } \chi \in \{0,1\}^{\leq D}$

# $i\mathcal{O}$ from FE: Traversal and Evaluation



# $i\mathcal{O}$ from FE

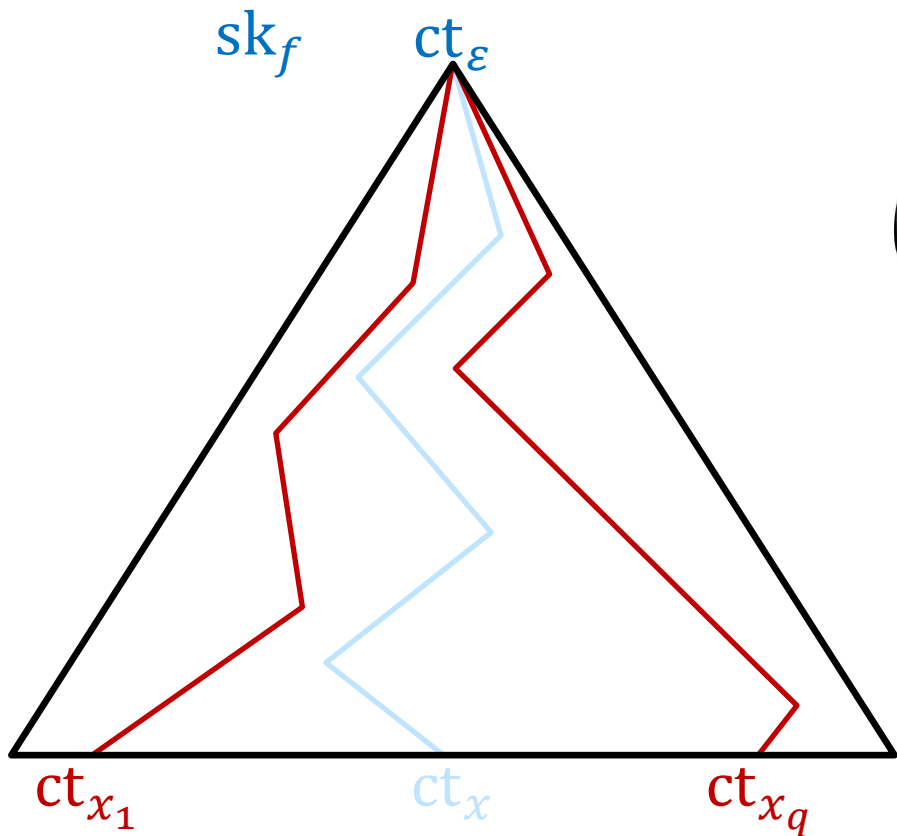
$$ct_{\chi}(C, \chi)$$

$$sk_f: (C, \chi) \mapsto (ct_{\chi \parallel 0}, ct_{\chi \parallel 1}) / C(\chi)$$

$sk_f$

$ct_{\varepsilon}$

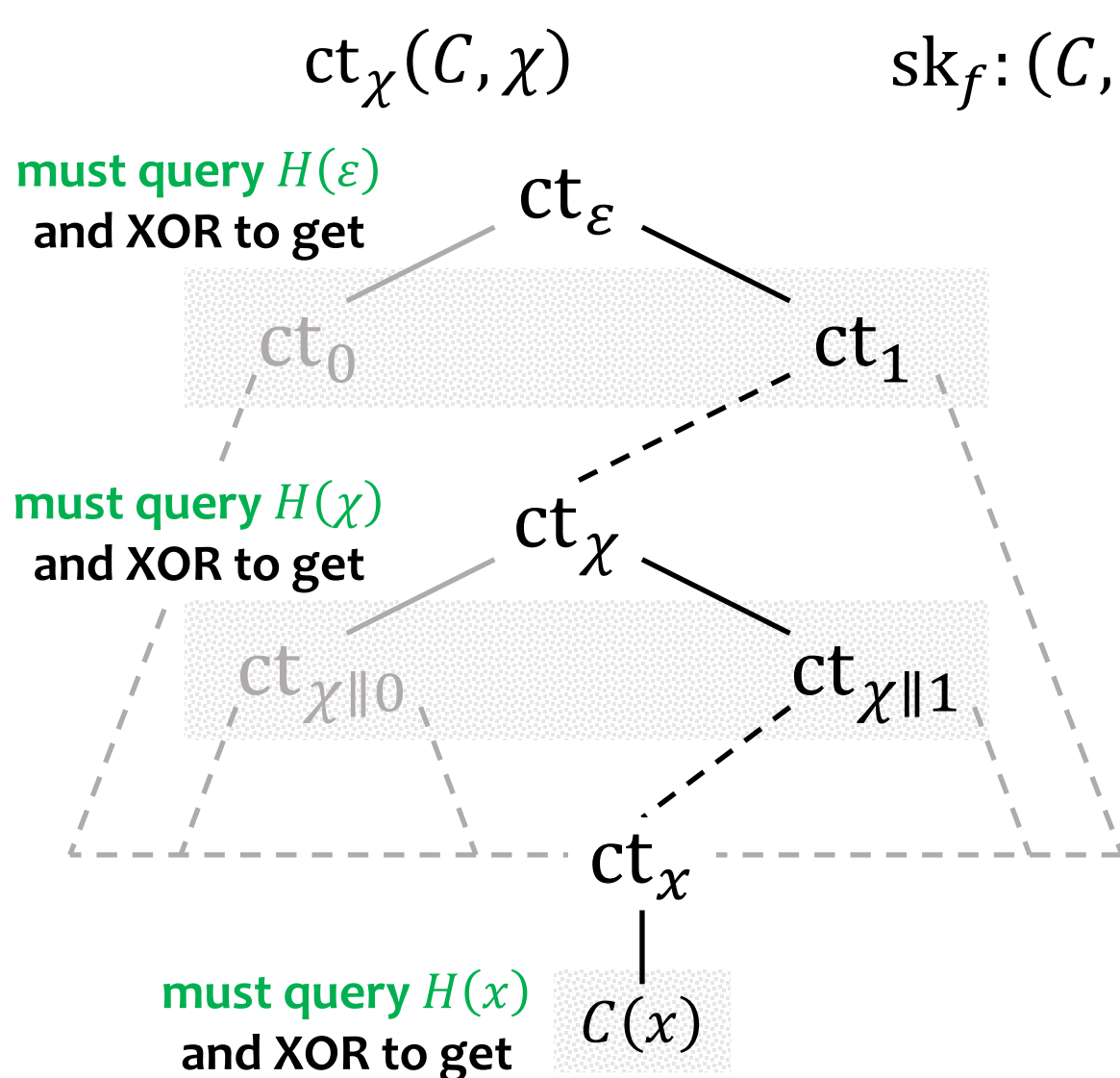
$$Obf(C) = (sk_f, ct_{\varepsilon})$$



← could evaluate at many inputs.  
Do **not know where** it explores.  
Reduction goes over **every** input.  
(**exponential loss**)

**Idealized models could help!**

# Simplified Idea in ROM



$$sk_f: (C, \chi) \mapsto \boxed{H(\chi)} \oplus ((ct_{\chi||0}, ct_{\chi||1}) / C(\chi))$$

**X cannot call RO in  
circuit sent to FE. KeyGen**

**FE. Dec( $sk_f, ct_\chi$ ) is random  
if  $H(\chi)$  is not queried**

**observe & program RO  
in simulation**

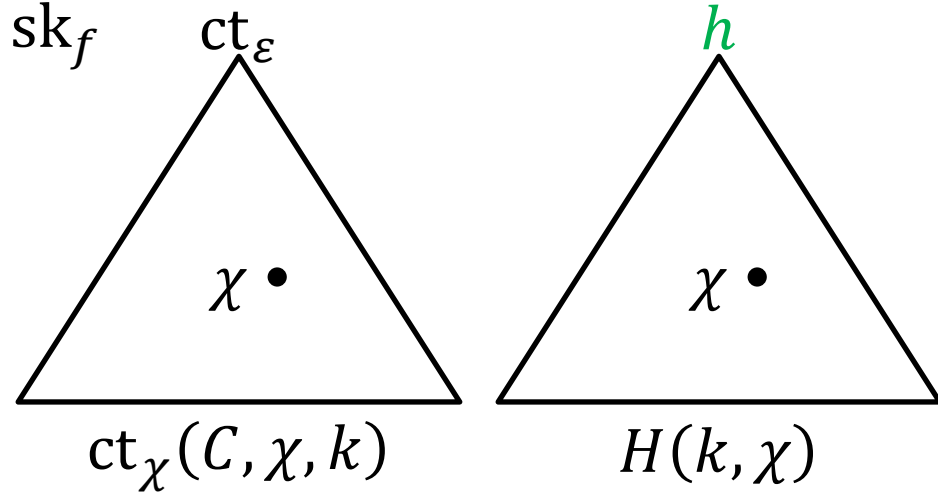


# First Attempt in PrOM

$$ct_\chi(C, \chi, k)$$

use code and  $k$

$$sk_f: (C, \chi, k) \mapsto H(k, \chi) \oplus ((ct_{\chi||0}, ct_{\chi||1})/C(\chi))$$

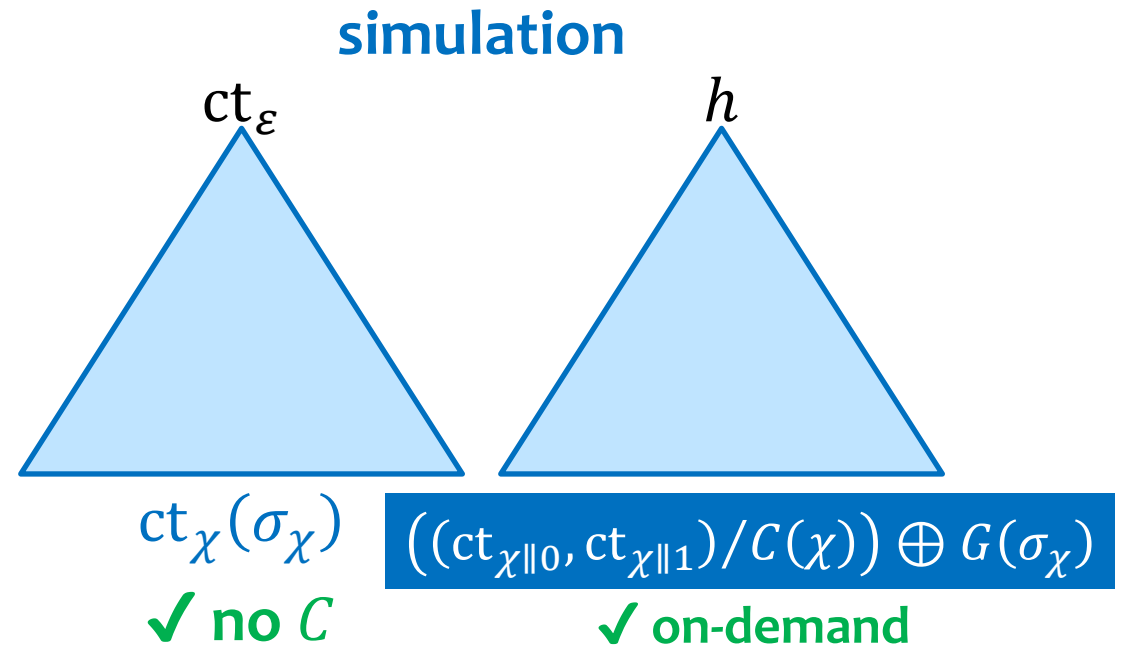
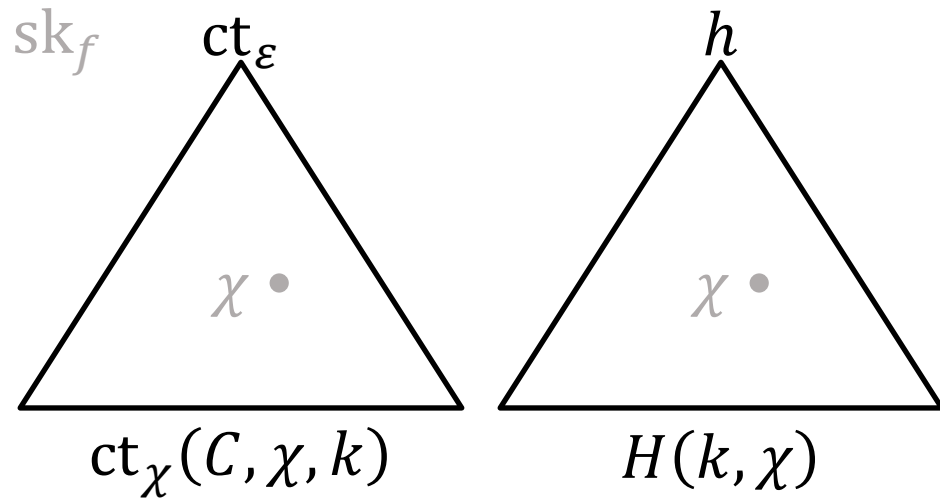


$$Dec(sk_f, ct_\chi) \oplus h(\chi) = (ct_{\chi||0}, ct_{\chi||1})/C(\chi)$$

call evaluation oracle with  $h$

# First Attempt: Simulation

$$\text{ct}_\chi \begin{cases} C, \chi, k \\ \boxed{\sigma_\chi} \text{ fresh seed} \\ \text{for each } \chi \end{cases} \quad \text{sk}_f \begin{cases} (C, \chi, k) \mapsto H(k, \chi) \oplus ((\text{ct}_{\chi||0}, \text{ct}_{\chi||1}) / C(\chi)) \\ \boxed{\sigma_\chi \mapsto G(\sigma_\chi)} \text{ PRG} \end{cases}$$



✓ **indistinguishable** { FE. Dec results, PrOM responses }:

$$\{H(k, \chi) \oplus (\dots), H(k, \chi)\}_{\chi \in \{0,1\}^{\leq D}} \approx \{G(\sigma_\chi), (\dots) \oplus G(\sigma_\chi)\}_{\chi \in \{0,1\}^{\leq D}}$$

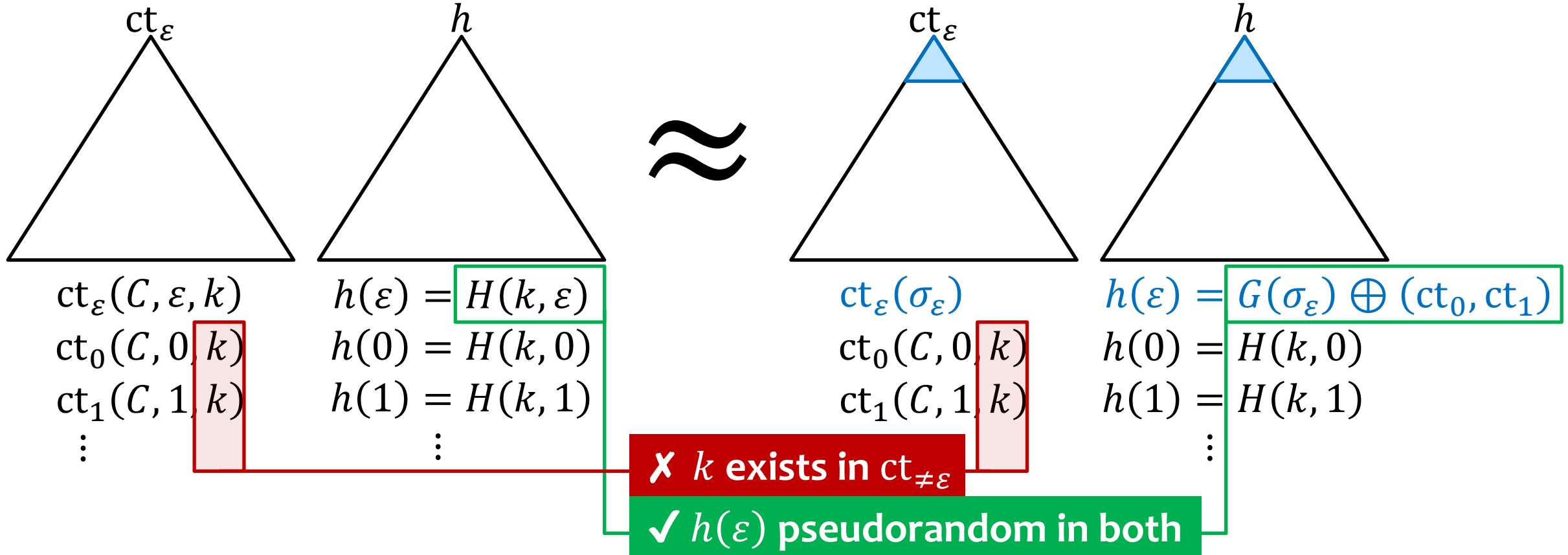
# First Attempt: Proof

1. separate functions for each level  
**NOT** making  $H$  puncturable PRF  
 (not good model of hash functions)

$$ct_\chi \begin{cases} C, \chi, k \\ \sigma_\chi \end{cases}$$

$$sk_f \begin{cases} H(k, \chi) \oplus (\dots) \\ G(\sigma_\chi) \end{cases}$$

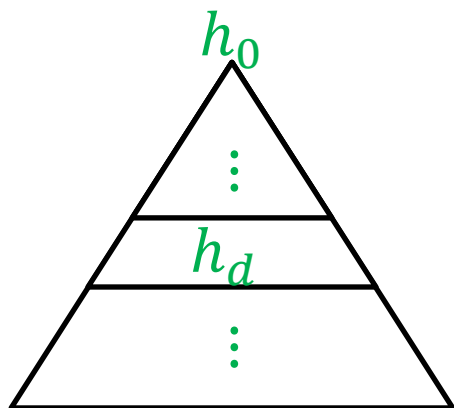
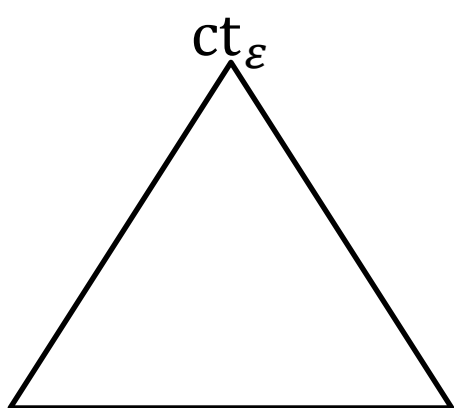
$$h \begin{cases} H(k, \chi) \\ G(\sigma_\chi) \oplus (\dots) \end{cases}$$



# Second Attempt

$$\text{ct}_\chi \begin{cases} C, \chi, k_{\geq |\chi|} \\ \sigma_\chi \end{cases}$$

$$\text{sk}_f \begin{cases} H(k_{|\chi|}, \chi) \oplus (\dots) \\ G(\sigma_\chi) \end{cases}$$



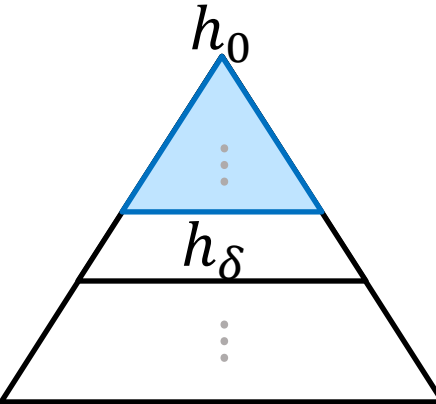
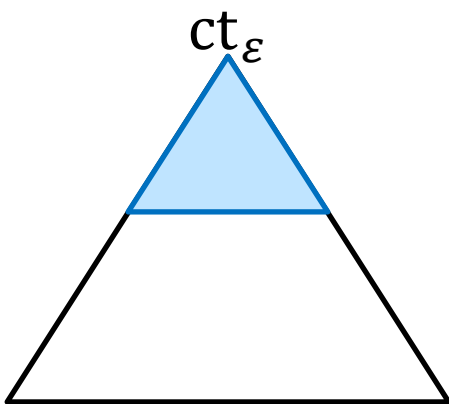
$$(\text{sim.}) h_d(\chi) \leftarrow \begin{cases} \$, & |\chi| \neq d; \\ G(\sigma_\chi) \oplus (\dots), & |\chi| = d. \end{cases}$$

# Second Attempt: Proof

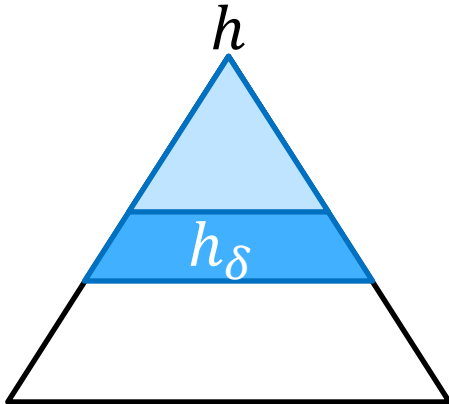
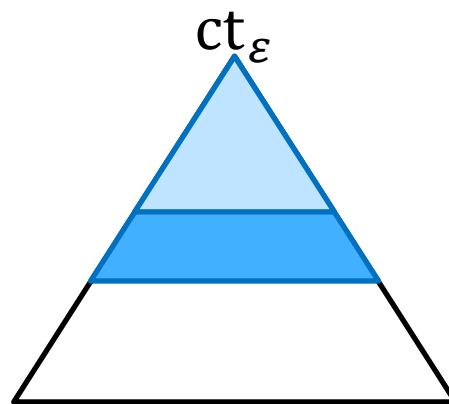
$$ct_{\chi} \begin{cases} C, \chi, k_{\geq |\chi|} \\ \sigma_{\chi} \end{cases}$$

$$sk_f \begin{cases} H(k_{|\chi|}, \chi) \oplus (\dots) \\ G(\sigma_{\chi}) \end{cases}$$

$$h_d \begin{cases} H(k_d, \chi) \\ \$/G(\sigma_{\chi}) \oplus (\dots) \end{cases}$$



≈



$ct_{|\chi| < \delta}(\sigma_{\chi})$   
 $ct_{|\chi| = \delta}(C, \dots)$   
 $ct_{|\chi| > \delta}(C, \dots)$

$h_{< \delta}: \$/\dots$   
 $h_{\delta}: H(k_{\delta}, \chi)$   
 $h_{d > \delta}: H(k_d, \chi)$

$ct_{|\chi| < \delta}(\sigma_{\chi})$   
 $ct_{|\chi| = \delta}(\sigma_{\chi})$   
 $ct_{|\chi| > \delta}(C, \dots)$

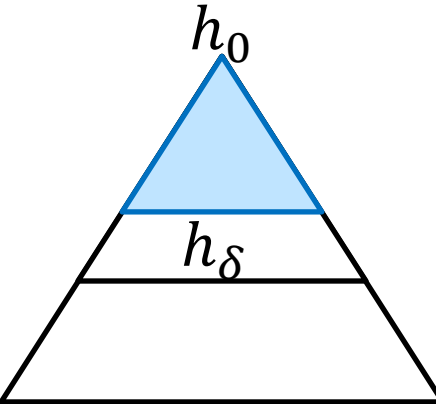
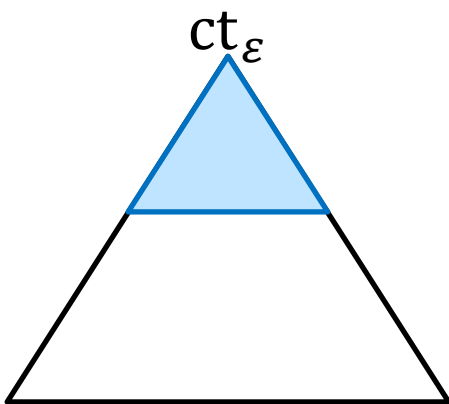
$h_{< \delta}: \$/\dots$   
 $h_{\delta}: \$/\dots$   
 $h_{d > \delta}: H(k_d, \chi)$

# Second Attempt: Hardwiring

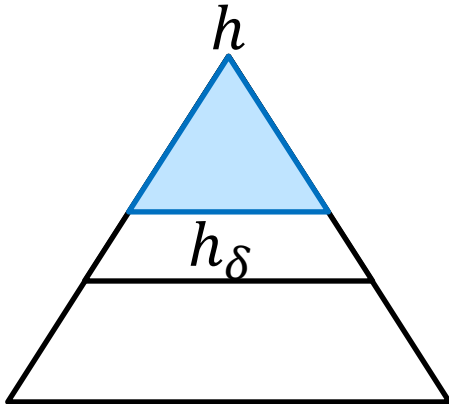
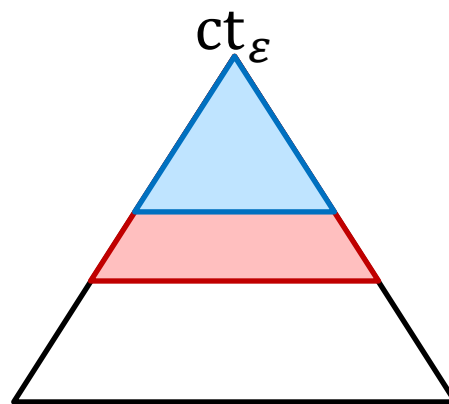
$$ct_x \begin{cases} C, \chi, k_{\geq |\chi|} \\ \sigma_x \end{cases}$$

$$sk_f \begin{cases} H(k_{|\chi|}, \chi) \oplus (\dots) \\ G(\sigma_x) \end{cases}$$

$$h_d \begin{cases} H(k_d, \chi) \\ \$/G(\sigma_x) \oplus (\dots) \end{cases}$$



≈



$ct_{|\chi| < \delta}(\sigma_x)$   
 $ct_{|\chi| = \delta}(C, \dots)$   
 $ct_{|\chi| > \delta}(C, \dots)$

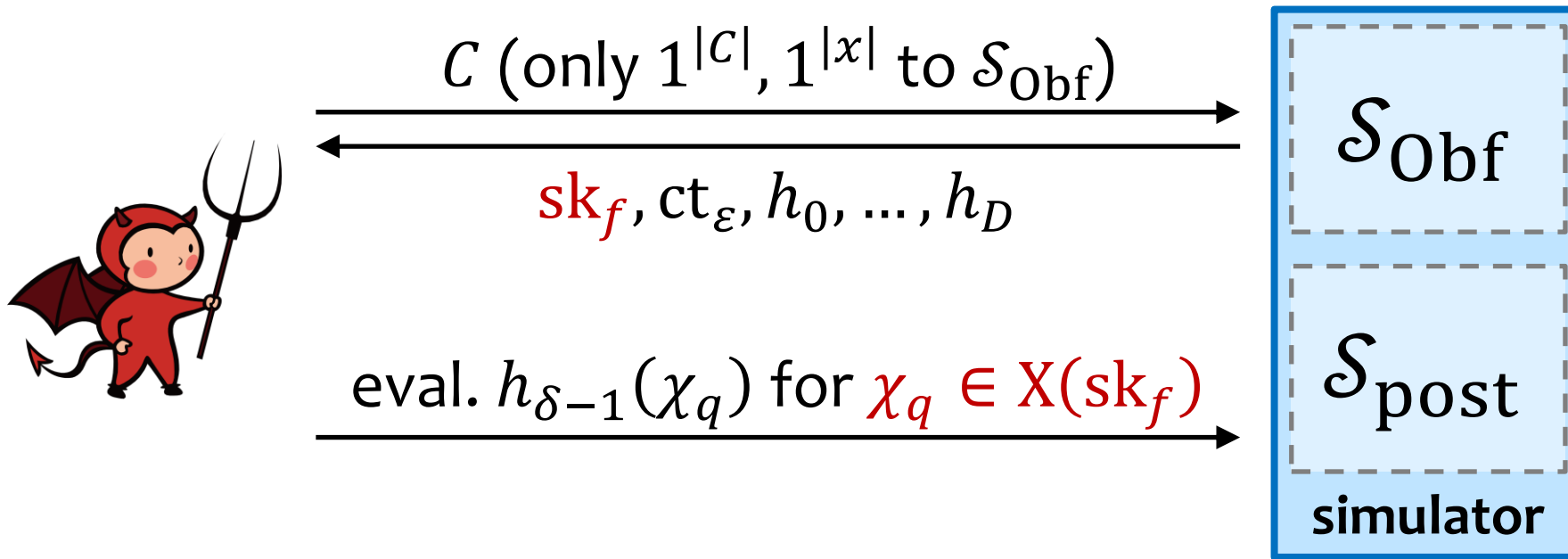
$h_{< \delta}: \$/\dots$   
 $h_{\delta}: H(k_{\delta}, \chi)$   
 $h_{d > \delta}: H(k_d, \chi)$

$ct_{|\chi| < \delta}(\sigma_x)$   
 $ct_{|\chi| = \delta} \mapsto H(k_{\delta}, \chi) \oplus (\dots)$   
 $ct_{|\chi| > \delta}(C, \dots)$

$h_{< \delta}: \$/\dots$   
 $h_{\delta}: H(k_{\delta}, \chi)$   
 $h_{d > \delta}: H(k_d, \chi)$

**Hardwire  $H(k_{\delta}, \chi) \oplus (\dots)$  into  $sk_f$ ?**

# Problem with Hardwiring into $sk_f$



querying  $h_{\delta-1}(\chi_q) = G(\sigma_{\chi_q}) \oplus (ct_{\chi_q \parallel 0}, ct_{\chi_q \parallel 1})$

forces  $\mathcal{S}_{\text{post}}$  to generate  $ct_{\chi_q \parallel 0}, ct_{\chi_q \parallel 1}$

put  $H(k_\delta, \chi) \oplus (\dots)$   
into  $ct_{|\chi|=\delta}$

1. **Cannot predict** (adaptively chosen)  $\chi_q$ 's dependent on  $sk_f$ .
2. **No space** to hardwire  $H(k_\delta, \chi_q \parallel 0) \oplus (\dots), H(k_\delta, \chi_q \parallel 1) \oplus (\dots)$  into  $sk_f$  for **all**  $\chi_q \in X(sk_f)$ .

# Problem with Hardwiring into $ct_\chi$

1. separate functions for each level
2. hardwire in ct of adaptive FE

$$ct_\chi \leftarrow \text{FE. Enc} \left( \text{mpk}, H(k_\delta, \chi) \oplus (ct_{\chi\parallel 0}, ct_{\chi\parallel 1}) \right)$$

**$|ct|$ -bit ciphertext**

**$2|ct|$ -bit plaintext**

**chop  $(ct_{\chi\parallel 0}, ct_{\chi\parallel 1})$  into blocks & hardwire one block at a time**

**$B = \Theta(\sqrt{|ct|})$  blocks & each block is  $B$  bits**

**OK if  $|ct| \leq |\text{plaintext}|^{2-\alpha}$**

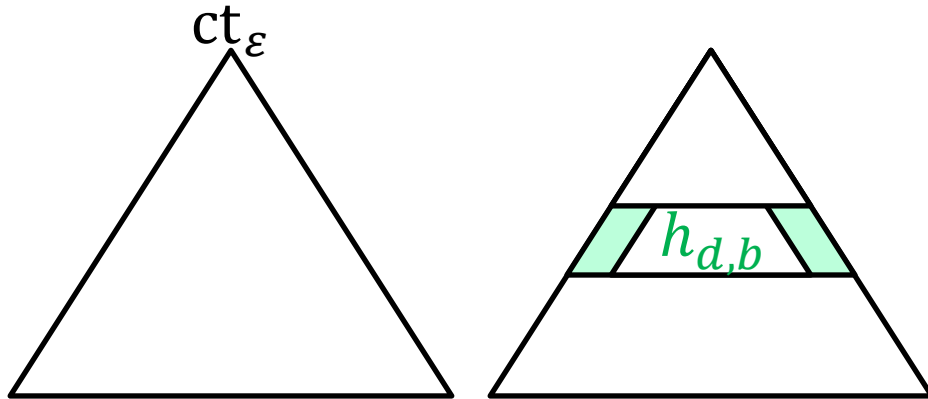


# Ideal Obfuscation in PrOM

1. separate functions for each level
2. hardwire in ct of adaptive FE
3. hardwire block by block

$$\text{ct}_\chi(C, \chi, \sigma_{\chi, \leq \beta}, k_{\geq |\chi|, > \beta}) \quad \text{sk}_f: G(\sigma_{\chi, \leq \beta}) \parallel (H(k_{|\chi|, > \beta}, \chi) \oplus (\dots))$$

stitching first  $\beta$  blocks (sim.) and last  $(B - \beta)$  blocks

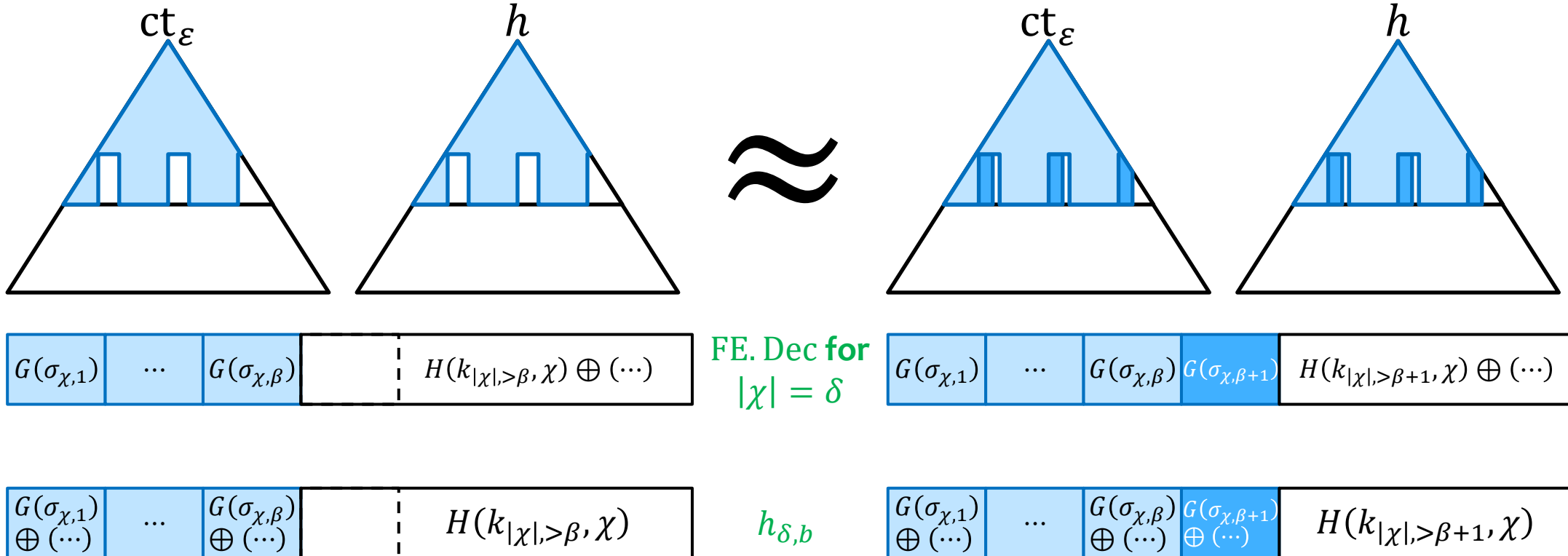


$$(\text{sim.}) h_{d,b}(\chi) \leftarrow \begin{cases} \$, & |\chi| \neq d; \\ G(\sigma_{\chi,b}) \oplus (\dots), & |\chi| = d. \end{cases}$$

# Ideal Obfuscation: Hybrid

1. separate functions for each level
2. hardwire in ct of adaptive FE
3. hardwire block by block

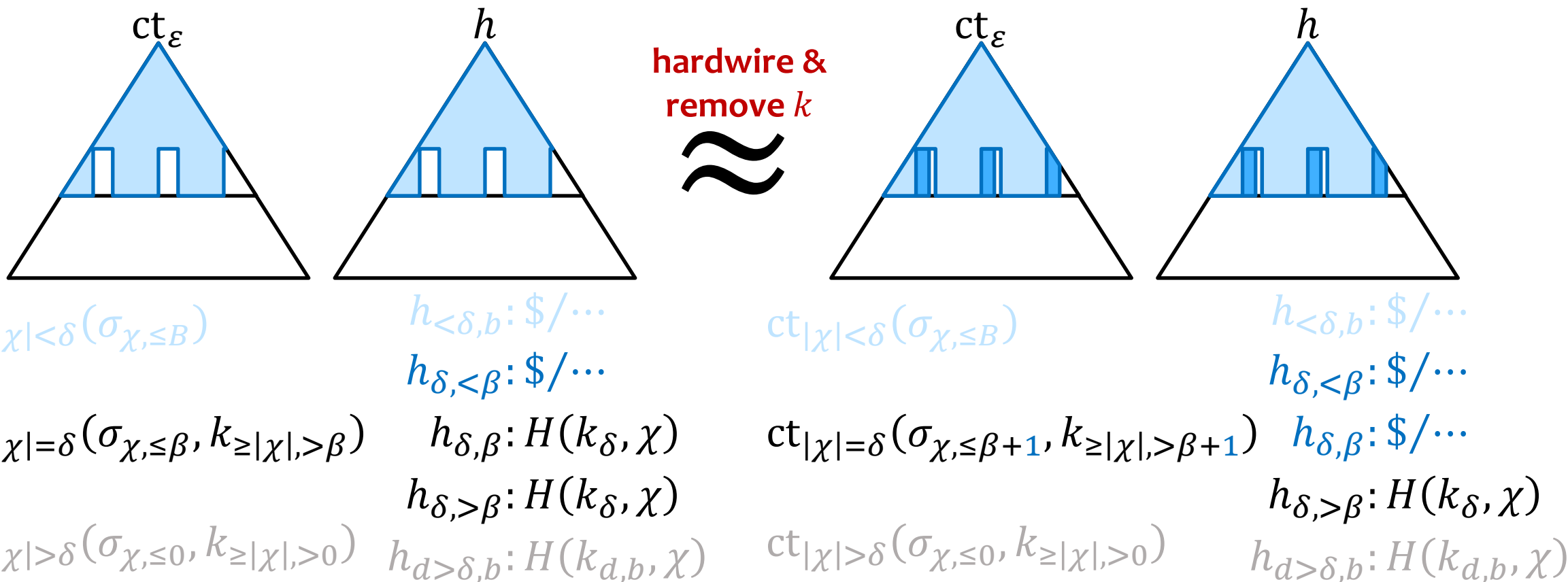
$$ct_{\chi}(C, \chi, \sigma_{\chi, \leq \beta}, k_{\geq |\chi|, > \beta}) \quad sk_f: G(\sigma_{\chi, \leq \beta}) \parallel (H(k_{|\chi|, > \beta}, \chi) \oplus (\dots)) \quad h_{d,b} \begin{cases} H(k_{d,b}, \chi) \\ \$/G(\sigma_{\chi, b}) \oplus (\dots) \end{cases}$$



# Ideal Obfuscation: Proof

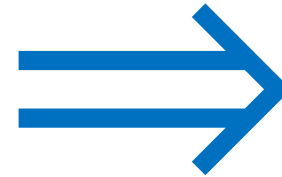
1. separate functions for each level
2. hardwire in ct of adaptive FE
3. hardwire block by block

$$ct_{\chi}(C, \chi, \sigma_{\chi, \leq \beta}, k_{\geq |\chi|, > \beta}) \quad sk_f: G(\sigma_{\chi, \leq \beta}) \parallel (H(k_{|\chi|, > \beta}, \chi) \oplus (\dots)) \quad h_{d,b} \begin{cases} H(k_{d,b}, \chi) \\ \$/G(\sigma_{\chi,b}) \oplus (\dots) \end{cases}$$



# Pseudorandom Oracle Model

(new model for random-looking  
hash functions with code)



# Ideal Obfuscation

**FE**

*Thank you!*

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